COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1943

IMPERIAL MYCOLOGICAL
2 JUN 1947
INSTITUTE

VOLUME XL

№ 1

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en régle générale quatre pages, sont, donnés en anglais ou en français. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 25 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

10 Juillet

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Arbusov, de l'Académie, **A. Borissiak**, de l'Académie, **V. Chlopin**, de l'Académie, **A. Doborin**, de l'Académie, **A. Fersman**, de l'Académie, **A. Frumkin**, de l'Académie, **A. Kolmogoroff**, de l'Académie (vice rédacteur), **L. Orbeli**, de l'Académie (rédacteur en chef), **A. Richter**, de l'Académie, **S. Soboleff**, de l'Académie, **S. Wawilow**, de l'Académie (vice rédacteur)

NOUVELLE SÉRIE

11-me année

Paraissant tous les dix jours

1943

VOLUME XL, № 1

TABLE DES MATIÈRES

MATHÉMATIQUES Pages

A. A. Kulakoff. Sur un critérium de non-simplicité d'un groupe fini 3
M. N. Olevsky. On a Generalization of Bessel Functions 5
N. Tschebotaröw, Corresponding Member of the Academy. On Representation of Lie Groups without Measure 11

MÉCANIQUE

A. M. Basin. A New Approximate Method for the Calculation of Laminar Boundary Layer 14

PHYSIQUE

M. A. Markov. On the Relativistically Invariant «Cutting-off Factor» in Electrodynamics 18

CHIMIE

P. P. Budnikov, Corresponding Member of the Academy, and **M. I. Strelkov.** On the Problem of Production of Alite Cement 20
G. V. Pigulevsky. Synthesis of Sulphanilamide (Streptocid album) 22
E. I. Speranskaya. Reciprocal System of Sodium and Barium Chlorides and Nitrates 23

CHIMIE DU GLOBE

Kh. G. Vinogradova. Presence of Molybdenum in *Leguminosae* 26

CHIMIE BIOLOGIQUE

V. L. Kretovich, K. V. Pshenova and **A. A. Bundel.** Proteolysis in Grain Affected with *Eurygaster integriceps* 30

HYDROBIOLOGIE

N. K. Decksbach. The Mollusc *Corbicula fluminalis* Müll. in the Valley of the Murgab River 33

ÉCOLOGIE

K. V. Arnoldi. On the Conditions and Phases of the Spring Transition of *Eurygaster integriceps* to Active Life as Observed in South-Western Uzbekistan 35

A. S. Kosmachevsky. Ecology and Distribution Areas of Some *Scarabaeidae* Species 38

Édition de l'Académie des Sciences de l'URSS
Moscou 1943

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

MATHÉMATIQUES

SUR UN CRITÉRIUM DE NON-SIMPLICITÉ D'UN GROUPE FINI

Par A. A. KULAKOFF

(Presenté par S. L. Soboleff, de l'Académie, le 25.III.1943)

Soit G un groupe abstrait d'ordre g, G'—un sous-groupe propre d'ordre g' de G*. Nous désignons par h_A le nombre des éléments de G, conjugués de A dans G. Le nombre des classes d'éléments conjugués de G soit r. Dans l'article présent on démontre le théorème suivant:

Si le central C de G' est distinct de 1 (de l'unité de G) et si, d'ailleurs, a lieu l'inégalité

$$g \sum_S \frac{1}{h_S} < (2r-1)\, g',$$

où S parcourt tous les éléments de G', alors G est un groupe composé.

Démonstration. Soient

$$\Gamma_1, \Gamma_2, \ldots, \Gamma_r$$

toutes les représentations irréductibles de G, Γ_1 étant la représentation dont toutes les matrices sont égales à 1. Désignons par $\Gamma_{u,\,G'}$ la représentation de G' contenue dans Γ_u, c'est-à-dire l'ensemble des matrices de Γ_u correspondant aux éléments de G'. Soit

$$\Gamma_{u,\,G'} = \sum_{v=1}^{r'} k_{uv}\, \gamma_v,$$

où r' est le nombre des classes d'éléments conjugués de G', et k_{uv} est le nombre indiquant combien de fois se reproduit la représentation irréductible γ_v de G' après la réduction complète de $\Gamma_{u,\,G'}$. On a

$$\sum_S \chi_S^u\, \chi_{S^{-1}}^u = g' \sum_{v=1}^{r'} k_{uv}^2 \qquad (1)$$

où χ_S^u est le caractère d'un élément S de G' dans Γ_u, et S parcourt tous les éléments de G'. On en déduit que la condition nécessaire et suffisante pour que $\Gamma_{u,\,G'}$ soit irréductible s'exprime par l'égalité

$$\sum_S \chi_S^u\, \chi_{S^{-1}}^u = g'.$$

* C'est-à-dire un sous-groupe distinct de G et de 1.

La formule (1) montre en outre que dans le cas où $\Gamma_{u,G'}$ est réductible, on a

$$\sum_S \chi_S^u \chi_{S^{-1}}^u \geqslant 2g'.$$

Nous allons démontrer maintenant qu'au moins pour un $u \geqslant 2$ le groupe $\Gamma_{u,G'}$ est irréductible. Considérons l'expression

$$\Omega = \sum_{u=1}^{r} \sum_S \chi_S^u \chi_{S^{-1}}^u,$$

où S parcourt tous les éléments de G'.

Si $\Gamma_{u,G'}$ était réductible pour chaque $u \geqslant 2$, on aurait

$$\Omega = g' + \sum_{u=2}^{r} \sum_S \chi_S^u \chi_{S^{-1}}^u \geqslant g' + 2(r-1)g' = (2r-1)g'.$$

Or, d'un autre côté, on a:

$$\Omega = \sum_S \sum_{u=1}^{r} \chi_S^u \chi_{S^{-1}}^u = \sum_S \frac{g}{h_S} = g \sum_S \frac{1}{h_S}.$$

On aurait donc:

$$g \sum_S \frac{1}{h_S} \geqslant (2r-1)g',$$

ce qui contredit notre hypothèse. Il existe donc un nombre $u' \geqslant 2$ pour lequel le groupe $\Gamma_{u',G'}$ est irréductible.

Soit maintenant R_C un élément quelconque de C distinct de 1. La matrice (R_C) qui correspond à R_C a la forme suivante

$$\begin{pmatrix} \alpha & 0 & \ldots & 0 \\ 0 & \alpha & 0 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & \alpha \end{pmatrix},$$

puisque (R_C) est permutable à toutes les matrices de la représentation irréductible $\Gamma_{u',G'}$. On en déduit aisément que le groupe G ne peut être simple. En effet, si le degré $f_{u'}$ de $\Gamma_{u'}$ est égal à 1, alors le nombre des représentations irréductibles de degré 1 du groupe G est plus grand que 1, d'où il suit que G est distinct de son commutant.

Supposons à présent que $f_{u'} > 1$. Alors $\Gamma_{u'}$ est non-abélien, car autrement $\Gamma_{u'}$ serait réductible. La matrice (R_C) ayant la forme ci-dessus indiquée appartient au central de $\Gamma_{u'}$. Si donc (R_C) n'est pas une matrice unité, $\Gamma_{u'}$ contient un diviseur normal propre à qui correspond un diviseur normal propre de G contenant R_C. Si (R_C) est une matrice unité, G est multiplement homomorphe à $\Gamma_{u'}$, donc G est composé. On peut noter encore qu'aussi dans ce cas il existe un diviseur normal propre de G contenant R_C. Le théorème est démontré.

Institut de Chemin de fer.
Tachkent.

Manuscrit reçu
le 15. II. 1943.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

MATHEMATICS

ON A GENERALIZATION OF BESSEL FUNCTIONS

By M. N. OLEVSKY

(Communicated by S. L. Soboleff, Member of the Academy, 28. III. 1943)

It has been shown by me (1) that the equation

$$\left(\frac{\sin\sqrt{k}\,s}{\sqrt{k}}\right)^2\frac{d^2S}{ds^2}+\frac{\sin\sqrt{ks}}{\sqrt{k}}\cos\sqrt{k}s\,\frac{dS}{ds}+\left[\mu^2\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^2-p^2\right]S=0 \qquad (1)$$

$$0\leqslant s\leqslant\frac{\pi}{\sqrt{k}},\quad k>0,$$

$$0\leqslant s<\infty,\quad k<0,$$

is of importance for many principal problems of the mathematical physics in a space of constant curvature k. This equation changes into the Bessel equation when $k\to 0$. The functions defined by it represent a far reaching generalization of Bessel functions.

In this paper some principal facts bearing on the theory of these functions are given.

1. By denoting $\frac{\sin\sqrt{ks}}{\sqrt{k}}=x$, $S(s)=y(x)$, we reduce (1) to the form

$$x^2(1-kx^2)\,y''+x(1-2kx^2)\,y'+(\mu^2x^2-p^2)\,y=0. \qquad (2)$$

The substitution $kx^2=\xi$, $y=\xi^{\frac{p}{2}}\psi(\xi)$ transforms (2) into a hypergeometric equation for ψ:

$$\xi(\xi-1)\frac{d^2\psi}{d\xi^2}+\left[\left(p+\frac{3}{2}\right)\xi-(p+1)\right]\frac{d\psi}{d\xi}+\frac{1}{4}\left[p(p+1)-\frac{\mu^2}{k}\right]\psi=0. \qquad (3)$$

If the function for $|x|<\frac{1}{\sqrt{|k|}}$ $\left(\text{or for } |x|>\frac{1}{\sqrt{|k|}}, \text{ its analytical extension}\right)$ be denoted by $J(k, p, \mu, x)$, the general solution of the equation (2) may be written in the form

$$y=C_1J(k, p, \mu, x)+C_2J(k, -p, \mu, x) \qquad (4)$$

where p is not an integer. If p is an integer, $J(k, p, \mu, x)$ and $J(k, -p, \mu, x)$ are linearly connected, so that the general solution takes the form

$$y=C_1J(k, p, \mu, x)+C_2Y(k, p, \mu, x), \qquad (4')$$

where $Y(k, p, \mu, x)$, generally speaking, has a logarithmical singularity at $x=0$ (2). When $k \to 0$ $J(k, p, \mu, x)$ [and $Y(k, p, \mu, x)$ if suitably normalized] changes into $J_p(\mu x)$ [and into $Y_p(\mu x)$, respectively], where $J_p(x)$ and $Y_p(x)$ are Bessel functions of order p of the 1st and of the 2nd kind. In the general case J and Y are not symmetrical with respect to μ and x; thus, we come here to a double generalization of Bessel functions (with respect to μ and with respect to x; cf. § 8, for instance).

The general solution of the initial equation (1) does not result from (4) when we formally change kx^2 into $\sin^2 \sqrt{k}s$, if at least $k > 0$, $\frac{\pi}{2\sqrt{k}} \leqslant s \leqslant \frac{\pi}{\sqrt{k}}$, since the derivative of $F\left(a, b, a+b+\frac{1}{2}, \sin^2 \sqrt{k}s\right)$ makes a jump at $s=\frac{\pi}{2\sqrt{k}}$ $(k>0)$ (2). It can be shown that the general solution of the equation (1) for $k>0$ and $0 \leqslant s \leqslant \frac{\pi}{\sqrt{k}}$ is to be written as follows *:

$$S(s)=C_1\left(\frac{\sin \sqrt{k}s}{\sqrt{k}}\right)^{-p} F\left(p+\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ p+\frac{1}{2}-\right.$$

$$\left.-\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ p+1,\ \sin^2 \frac{\sqrt{k}}{2} s\right)+C_2\left(\frac{\sin \sqrt{k}s}{\sqrt{k}}\right)^{-p} \times$$

$$\times F\left(-p+\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ -p+\frac{1}{2}-\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ -p+1,\ \sin^2 \frac{\sqrt{k}}{2} s\right) \quad (4'')$$

(p is not an integer).

If $k=-\alpha^2<0$ and s takes real values, the general solution of (1) can as well remain in the form (4) (when $\operatorname{sh}^2 \alpha s < 1$), $-\operatorname{sh}^2 \alpha s$ being substituted for kx^2.

Now we denote the function

$$\frac{\mu^p}{2^p \Gamma(p+1)}\left(\frac{\sin \sqrt{k}s}{\sqrt{k}}\right)^p F\left(p+\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\right.$$

$$\left.p+\frac{1}{2}-\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ p+1,\ \sin^2 \frac{\sqrt{k}}{2} s\right)$$

by $J_p^+(\mu, s)$ for $k=\alpha^2>0$ and by $J_p^-(\mu, s)$ for $k=-\alpha^2<0$. When the same conclusions can be made for $k>0$ and $k<0$, this function is denoted by $J_p(\mu, s)$, whereby its dependence on k is omitted. In the following, for the sake of brevity, only that function $J_p(\mu, s)$ which generalizes the Bessel function of the 1st kind [for $k \to 0$ it changes into $J_p(\mu, s)$] will be considered, and will mainly be considered as a function of s.

---

* A particular consequence of this is that for arbitrary μ and $p \geqslant 0$, $k>0$, there is no solution of (1) different from zero and regular at $s=0$ and $s=\frac{\pi}{\sqrt{k}}$, since for

$$F\left(a, b, c, \sin^2 \frac{\sqrt{k}}{2} s\right) \equiv F\left(p+\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ p+\right.$$

$$\left.+\frac{1}{2}-\frac{1}{2}\sqrt{1+\frac{4\mu^2}{k}},\ p+1,\ \sin^2 \frac{\sqrt{k}}{2} s\right)$$

the equality $c-a-b=-p$ holds (2).

2. The following recurrent formulae ([3])

$$\left.\begin{aligned}
&\prod_{i=0}^{m-1}\left[k(p+i)(p+i+1)-\mu^2\right]\frac{J_{p+m}(\mu,s)}{\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^{p+m}}=\\
&\qquad=\mu^m\left(\frac{\sqrt{k}}{\sin\sqrt{k}s}\frac{d}{ds}\right)^m\frac{J_p(\mu,s)}{\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^p},\\
&\left(\frac{\sqrt{k}}{\sin\sqrt{k}s}\frac{d}{ds}\right)^m\left[\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^p J_p(\mu,s)\right]=\mu^m\left(\frac{\sin\sqrt{k}s}{\sqrt{k}}\right)^{p-m}J_{p-m}(\mu,s),\\
&2p\sqrt{k}\operatorname{ctg}\sqrt{k}s\,J_p(\mu,s)=\mu\left[J_{p-1}(\mu,s)+J_{p+1}(\mu,s)\right]-\\
&\qquad-\frac{kp(p+1)}{\mu}J_{p+1}(\mu,s)
\end{aligned}\right\}\quad(5)$$

hold, where $\left(\frac{\sqrt{k}}{\sin\sqrt{k}s}\frac{d}{ds}\right)^m$ denotes the operation $\frac{\sqrt{k}}{\sin\sqrt{k}s}\frac{d}{ds}$ repeated m times.

3. $J_p(\mu,s)$ degenerates into an elementary function when $p=\pm\left(m-\frac{1}{2}\right)$ (m being a positive integer) ([3]), and when $\mu^2=k(p+N)(p+N+1)$, N being a non-negative integer.

4. $J_p(\mu,s)$, as a function of s, has an infinite number of real roots. When $\mu^2+\alpha^2p(p+1)>0$, the roots of $J_p(\mu,s)$ and $J_{p+1}(\mu,s)$ are interlacing ([3]).

5. The formula ([3])

$$J_p(\mu,s)\frac{dJ_{-p}(\mu,s)}{ds}-J_{-p}(\mu,s)\frac{dJ_p(\mu,s)}{ds}=\frac{2}{\pi}\sqrt{k}\operatorname{ctg}\sqrt{k}s\sin p\pi\qquad(6)$$

holds; hence follows a linear dependence between $J_p(\mu,s)$ and $J_{-p}(\mu,s)$, p being an integer.

6. When $p>-1$, $\mu\neq\nu$ and $\omega<\frac{\pi}{\sqrt{k}}$ (for $k>0$) (or, respectively, $\omega<\infty$, for $k<0$) we have ([3])

$$\begin{aligned}
&\int_0^\omega\frac{\sin\sqrt{k}s}{\sqrt{k}}J_p(\mu,s)J_p(\nu,s)=\qquad(7)\\
&=\frac{1}{\mu^2-\nu^2}\left\{\frac{\mu^2-kp(p+1)}{\mu}J_p(\nu,\omega)J_{p+1}(\mu,\omega)-\right.\\
&\left.-\frac{\nu^2-kp(p+1)}{\nu}J_p(\mu,\omega)J_{p+1}(\nu,\omega)\right\}\frac{\sin\sqrt{k}\omega}{\sqrt{k}}.
\end{aligned}$$

7. With $\mu^2>\frac{\alpha^2}{4}$ $J_p(\mu,s)$ admits the following asymptotic representation ([3]):

$$J_p(\mu,s)=\frac{A(\mu,p)\cos\sqrt{\mu^2-\frac{\alpha^2}{4}}s+B(\mu,p)\sin\sqrt{\mu^2-\frac{\alpha^2}{4}}s}{\sqrt{\frac{\operatorname{sh}\alpha s}{\alpha}}}+O\left(\left(\frac{\operatorname{sh}\alpha s}{\alpha}\right)^{-\frac{5}{2}}\right).\qquad(8)$$

For lack of space the expressions for $A(\mu,p)$ and $B(\mu,p)$ are here omitted. The order of growth of $J_p^+(\mu,s)$ with $s\to\frac{\pi}{\sqrt{k}}$ can likewise be evaluated.

8. The spectral expansion by the functions $J_p(\mu,s)\left(\mu^2>\frac{\alpha^2}{4}\right)$ [generalizations of the Fourier-Hankel formula ([3])]. If $R(p)>-1$ and $f(x)$ is such that

1) the conditions of Dirichlet are fulfilled within any finite interval,

2′) either $\int_{\alpha/2}^{\infty} K_p(x) f(x)\, dx$ is absolutely convergent,

2″) or $\int_0^{\infty} \frac{\operatorname{sh} \alpha x}{\alpha} f(x)\, dx$ is absolutely convergent,

where $K_p(x) = \frac{\alpha^{2p}}{\pi x^{2p-1}} \operatorname{sh} \frac{\pi \sqrt{x^2 - \frac{\alpha^2}{4}}}{\alpha} \Gamma\left(p + \frac{1}{2} + \frac{i}{\alpha}\sqrt{x^2 - \frac{\alpha^2}{4}}\right) \times$

$$\times \Gamma\left(p + \frac{1}{2} - \frac{i}{\alpha}\sqrt{x^2 - \frac{\alpha^2}{4}}\right),$$

then either

$$f(x) = \int_0^{\infty} \frac{\operatorname{sh} \alpha t}{\alpha} J_p^-(x, t)\, \varphi(t)\, dt, \qquad \varphi(t) = \int_{\frac{\alpha}{2}}^{\infty} K_p(z)\, J_p^-(z, t)\, f(z)\, dz, \tag{9′}$$

or, respectively,

$$f(x) = \int_{\frac{\alpha}{2}}^{\infty} K_p(t)\, J_p^-(t, x)\, \varphi(t)\, dt, \qquad \varphi(t) = \int_0^{\infty} \frac{\operatorname{sh} \alpha z}{\alpha} J_p^-(t, z)\, f(z)\, dz. \tag{9″}$$

The conditions 1) and 2) are sufficient. At the discontinuity points of $f(x)$ the left-hand side of (9′) is to be replaced by $\frac{f(x-0) + f(x+0)}{2}$.

The spectral representation by the functions $J_p^-(\mu, s)$ for integer and half-integer values of p are the best fitted for exhaustive use; here $K_p(x)$ changes into

$$K_m(x) = \frac{\prod_{i=0}^{m} [x^2 + (i-1)\, i\alpha^2]}{x^{2m+1}} \operatorname{th} \frac{\pi \sqrt{x^2 - \frac{\alpha^2}{4}}}{\alpha},$$

$$K_{m+\frac{1}{2}}(x) = \frac{\prod_{i=0}^{m} \left[x^2 + \left(i - \frac{1}{2}\right)\left(i + \frac{1}{2}\right)\alpha^2\right]}{x^{2m} \sqrt{x^2 - \frac{\alpha^2}{4}}},$$

where m is a non-negative integer.

9. Let us note some important «Eigenwert» problems connected with the equation (1). For different splitting processes leading to the equation (1) a finite or a vanishing solution may be required. Thus, in case $k > 0$ the equation (1) is connected with the problem a) of finding the solutions of (1) bounded at the endpoint of the interval $\left(0, \frac{2}{\sqrt{k}}\right)$. For $p \neq 0$ the Green function corresponding to this problem is written as follows:

$$G(s, \sigma) = \frac{1}{2p}\left(\operatorname{tg} \frac{\sqrt{k}}{2} s \operatorname{ctg} \frac{\sqrt{k}}{2} \sigma\right)^p \qquad s \leqslant \sigma$$

$$= \frac{1}{2p}\left(\operatorname{tg} \frac{\sqrt{k}}{2} \sigma \operatorname{ctg} \frac{\sqrt{k}}{2} s\right)^p \qquad s \geqslant \sigma.$$

Every function $f(s)$ bounded at the endpoints of the interval $\left(0, \frac{\pi}{\sqrt{k}}\right)$, with a continuous first derivative and a stepwise continuous second derivative, can be expanded in an absolutely and uniformly convergent series by the «Eigenfunktionen» of the problem:

$$f(s) = \sum_{N=0}^{\infty} c_N S_N(s) \qquad (10)$$

where $S_N(s) = J_p^+(\mu_N, s)$; $\mu_N^2 = k(p+N)(p+N+1)$; $N = 0, 1, 2, \ldots$ (see the Remark in p. 1), and

$$\int_0^{\frac{\pi}{\sqrt{k}}} \frac{\sin \sqrt{k}s}{\sqrt{k}} J_p^+(\mu_N, s) J_p^+(\mu_{N'}, s)\, ds = 0 \quad (N \neq N'),$$

consequently

$$c_N = \frac{\int_0^{\pi/\sqrt{k}} \frac{\sin \sqrt{k}s}{\sqrt{k}} J_p^+(\mu_N, s) f(s)\, ds}{\int_0^{\pi/\sqrt{k}} \frac{\sin \sqrt{k}s}{\sqrt{k}} J_p^+(\mu_N, s)\, ds}.$$

b) In case $k < 0$ the corresponding problem can be formulated as follows: to find the solutions of (1) which are finite at $s = 0$ and $s = \infty$. In this case, when $p > -1$, the spectrum is continuous, namely, consisting of all μ, satisfying $\mu^2 \geqslant -\frac{\alpha^2}{4}$; the «Eigenfunktionen» are $J_p(\mu, s)$, and the theorem about the expansion (10) changes into the spectral representation (9″) for $f(s)$.

c) A practical importance is attached further to the problem of finding the solutions of (1) ($k \lesseqgtr 0$) finite at $s = 0$ and vanishing at $s = R$. In this case too, the Green function can be written explicitly ([3]):

$$G(s, \sigma) = \frac{1}{2p}\left[\left(\operatorname{ctg}\frac{\sqrt{k}}{2}\sigma\right)^p - \left(\operatorname{tg}\frac{\sqrt{k}}{2}\sigma\right)^p \times \right.$$
$$\left. \times \left(\operatorname{ctg}\frac{\sqrt{k}}{2}R\right)^{2p}\right]\left(\operatorname{tg}\frac{\sqrt{k}}{2}s\right)^p \quad s \leqslant \sigma$$
$$= \frac{1}{2p}\left[\left(\operatorname{ctg}\frac{\sqrt{k}}{2}s\right)^p - \left(\operatorname{tg}\frac{\sqrt{k}}{2}s\right)^p \times \right.$$
$$\left. \times \left(\operatorname{ctg}\frac{\sqrt{k}}{2}R\right)^{2p}\right]\left(\operatorname{tg}\frac{\sqrt{k}}{2}\sigma\right)^p \quad s \geqslant \sigma$$

From this, as in problem a), there follows the existence of a discrete spectrum of «Eigenwerte» $\{\mu_N\}$ [which are roots of the equation $J_p(\mu, R) = 0$], the orthogonality of «Eigenfunktionen» in the interval $(0, R)$ (with the weight $\frac{\sin \sqrt{k}s}{\sqrt{k}}$) and the corresponding development by the functions $J_p(\mu_N, s)$.

For $k \to 0$ all formulae and propositions change into well-known facts from the theory of Bessel functions*.

In the following I intend to publish the proofs of these propositions as well as to apply them to the wave equation and to the equation of the conduction of heat in an n-dimensional space of constant curvature.

Pedagogical Institute.
Kzyl-Orda.

Received
25. III. 1943.

REFERENCES

[1] M. Olevsky, C. R. Acad. Sci. URSS, XXXIII, No. 4 (1941). [2] E. T. Whittaker and G. N. Watson, A Course of Modern Analysis, 4 ed., ch. 14, 15. [3] R. Courant—D. Hilbert, Methoden der Math. Physik, I, Kap. 5, § 15, Kap. 7, § 2 (7, 8), § 6(2). [4] G. N. Watson, A Treatise of the Theory of Bessel Functions, 406, 1922.

* Except for the order of vanishing of the difference

$$J_p^-(\mu, s) - \frac{C(\mu, p) \sin\left[\sqrt{\mu^2 - \frac{\alpha^2}{4}}\, s + \varphi(\mu, p)\right]}{\sqrt{\frac{\operatorname{sh} \alpha s}{\alpha}}}$$ (cf. p. 7).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

MATHEMATICS

ON REPRESENTATION OF LIE GROUPS WITHOUT MEASURE

By N. TSCHEBOTARÖW, Corresponding Member of the Academy

This paper has arisen from a conversation with I. M. Gelfand about the possibility to extend the notion of measure of Lie groups to representations different from simply transitive (i. e. from so-called parametric groups). The notion of measure (or integral invariant) ([5]) can be carried to a representation of a Lie group given by infinitesimal operators

$$X_i(f) = \xi_{iv} \frac{\partial f}{\partial x_v} \qquad (i = 1,2, \ldots, r).$$

in a following manner. By a measure of such a representation we mean the integral

$$\int\int \ldots \int f(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \ldots dx_n,$$

where f (the density of the representation) is a solution of the partial differential system

$$X_i(f) + \sigma_i f = 0 \quad (i = 1,2, \ldots, r), \tag{1}$$

where

$$\sigma_i = \frac{\partial \xi_{i1}}{\partial x_1} + \frac{\partial \xi_{i2}}{\partial x_2} + \ldots + \frac{\partial \xi_{in}}{\partial x_n}. \tag{2}$$

Aside from the classical papers of Lie ([3]), the question as to the existence of the measure was discussed in a few recent papers([1], [2], [4]) from an abstract point of view. The aim of this paper is to obtain the conditions for the existence of the measure in terms of the classical (local) theory of Lie groups.

If we look for a solution of the system (1) in the implicit form

$$\Phi(x_1, x_2, \ldots, x_n;\ f) = \text{const},$$

then Φ is a solution of the homogeneous system

$$X_i(\Phi) - \sigma_i f \frac{\partial \Phi}{\partial f} = 0 \quad (i = 1,2, \ldots, r). \tag{3}$$

Thus, this representation of the group has a measure if and only if the system (3) has a solution which really contains f.

Let us recall some facts which are either well known or obvious.

1. If the representation is transitive, the system (3) has one solution at most.

For, otherwise, the system (3) should have a solution not containing f. Such a solution would satisfy the system

$$X_i(\Phi) = 0 \qquad (i = 1, 2, \ldots, r),$$

which is impossible because the representation is transitive.

II. If the representation is simply transitive, i. e. if $n=r$, it has a measure.

For, if we introduce the notation

$$\overline{X}_i(\Phi)=X_i(\Phi)-\sigma_i f\frac{\partial\Phi}{\partial f}\qquad(i=1,2,\ldots,r),\tag{4}$$

then from the structure relations

$$(X_i,\ X_j)=c_{ij}^{v}X_v,$$

we obtain

$$(\overline{X}_i,\ \overline{X}_j)=c_{ij}^{v}\overline{X}_v,$$

whence follows that the system (3) is complete.

III. If

$$\sigma_i=0\qquad(i=1,2,\ldots,r),$$

the representation has a measure.

In this case $\Phi=f$ is a solution of the system (3). Putting $f=\text{const}$, we obtain the measure in the form

$$\int\int\ldots\int dx_1\,dx_2\ldots dx_n.$$

Each transitive representation of a Lie group G is defined by its subgroup H which maps as a stationary subgroup in this representation. The order of H is $r-n$. The representation of H is generated by such operators X_i that are of order $\geqslant 1$ with respect to some point of general position, which can without loss of generality be taken for the origin:

$$P_0\,(0,\ 0,\ \ldots,\ 0).$$

Let

$$Y_i\,(\Phi)\qquad(i=n+1,\ \ldots,\ r)$$

be the generating operators of the stationary subgroup H. If our representation does not have a measure, then the system (3) has no solution, and therefore the operators (4) produce a transitive representation of G in the $(n+1)$-dimensional space of the variables $x_1,\ x_2,\ \ldots,\ x_n,\ f$. In this representation the operators which correspond to H are

$$\overline{Y}_i\,(\Phi)=Y_i\,(\Phi)-\tau_i f\frac{\partial\Phi}{\partial f}\qquad(i=n+1,\ \ldots,\ r).\tag{5}$$

Since the term with $\frac{\partial\Phi}{\partial f}$ in these operators has f as factor, the point

$$x_1=x_2=\ \ldots\ =x_n=0,\ f=0$$

cannot be considered as a point of general position (the rank of the matrix

$$\|\xi_{i1}\,(x_1,\ \ldots,\ x_n),\ \xi_{i2},\ \ldots,\ \xi_{in},\ \sigma_i f\|$$

for this point is less than for a point of general position).

Now the point

$$x_1=x_2=\ \ldots\ =x_n=0,\quad f=\alpha\neq 0$$

is a point of general position. The stationary subgroups of G in the new representation (which will be denoted by H_1) must be of order $r-n-1$. Its operators must be linear combinations of the operators (5), because no linear combination of the remaining operators $X_i(\Phi)$ [and still less so of $X_i(\Phi)$] is an operator of order $\geqslant 1$. Hence follows that H_1 is a subgroup of H of order 1 with respect to H.

Thus, there are exactly $n-r-1$ independent operators of order $\geqslant 1$ among the operators (5). But the form of the operators (5) shows that an operator (5) is of order $\geqslant 1$ if and only if the function τ_i vanishes at $x_1 = x_2 = \ldots = x_n = 0$.

Now we suppose that for a given representation of a group G one of the functions τ_i, say τ_1, does not vanish at P_0. Then the operator $\overline{Y}_1(\Phi)$ is of order zero with respect to the point

$$x_1 = x_2 = \ldots = x_n = 0, \quad f = \alpha.$$

Hence follows that the representation (4) of the group G contains $n+1$ independent operators whose linear combinations cannot possibly be of order $\geqslant 1$. Thus, the representation (4) is transitive, the system (3) has no solution, and we obtain:

IV. A transitive representation of a Lie group has a measure if and only if the functions σ_i which correspond to the operators of order $\geqslant 1$ vanish at $x_1 = x_2 = \ldots = x_n = 0$.

The derived operator of two operators (5) of the group H has the form

$$(\overline{Y}_i, \overline{Y}_j) = (Y_i, Y_j) - Y_i(\tau_j) f \frac{\partial \Phi}{\partial f} + Y_j(\tau_i) f \frac{\partial \Phi}{\partial f}.$$

Since the operators Y_i, Y_j are of order $\geqslant 1$, the functions $Y_i(\tau_j)$ and $Y_j(\tau_i)$ vanish at $x_1 = x_2 = \ldots = x_n = 0$. Therefore the operator $(\overline{Y}_i, \overline{Y}_j)$ is of order $\geqslant 1$, and consequently enters into H_1; hence follows:

V. H_1 is an invariant subgroup of H. The function $\bar{\sigma}_i$ for an operator (4) of the group G has the value:

$$\bar{\sigma}_i = \frac{\partial \xi_{i1}}{\partial x_1} + \frac{\partial \xi_{i2}}{\partial x_2} + \ldots + \frac{\partial \xi_{in}}{\partial x_n} + \frac{\partial(-\sigma_i f)}{\partial f} = \sigma_i - \sigma_i = 0.$$

Hence follows that the representation (4) of the group G

$$\sigma_i = 0 \qquad (i = 1, 2, \ldots, r)$$

holds, so that this representation, in virtue of III, has a measure.

Finally, it may be remarked that $f = 0$ is an invariant manifold of the representation (4) of the group G.

Summarizing these facts gives the following conditions that are necessary for a transitive representation to have no measure:

VI. If a n-dimensional transitive representation of a group G has no measure, then the corresponding stationary subgroup H has an invariant subgroup H_1 of index 1. The $(n+1)$-dimensional representation of G corresponding to the stationary subgroup H_1 is transitive and has an invariant manifold of dimension n.

Example. The representation

$$X_1(f) = \frac{df}{dx}, \quad X_2(f) = x\frac{df}{dx}, \quad X_3(f) = x^2\frac{df}{dx}$$

of dimension 1 has no measure, because it contains the operator $X_2(f)$ of order 1 whose corresponding function $\sigma_2 = 1$ does not vanish at the point of general position $x = 0$.

Received
12. IV. 1943.

REFERENCES

[1] S. Banach, Monogr. Mat., 2, Warszawa (1923). [2] A. Haar, Ann. of Math. (2), 34, 147 (1933). [3] S. Lie, Ber. Sächs. Ges., Lpz., 342 (1897). [4] J. V. Neumann, Comp. Math., 1 (1934). [5] Н. Чеботарев, Теория групп Ли, 1940.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

MECHANICS

A NEW APPROXIMATE METHOD FOR THE CALCULATION OF LAMINAR BOUNDARY LAYER

By A. M. BASIN

(Communicated by N. E. Kochin, Member of the Academy, 3. III.1943)

For the confuser region, i. e. for the one situated between the frontal critical point and the point of minimum pressure (maximum of internal velocity), the problem of velocity distribution and friction in the region of laminar boundary layer has a comparatively simple solution. The greatest difficulties presents the diffuser region, i. e. that beyond the point of minimum pressure, where the fluid in the internal flow slows down its movement.

While the method of Kárman-Pohlhausen [1], which is the most extensively used and simple method for the calculation of the laminar boundary layer, gives satisfactory agreement with the experiment in the confuser region, it leads to considerable deviation from experimental data in the diffuser region, especially so in the region of separation.

In the present paper another method is suggested for an approximate calculation of the laminar boundary layer on a wing or on a body of revolution. It is as simple as that of Kárman-Pohlhausen and by using it satisfactory results are obtained not only for the whole of the diffuser region, but also for the confuser region near the minimum of pressure.

1. Let x and y by the coordinates of the boundary layer points measured along the stream-line profile and along the normal to the latter, respectively. Denote further by $u(x, y)$ the longitudinal velocity and by $U(x, \delta)$ the velocity on the outer border of the layer $y = \delta(x)$.

The function $u(x, y)$ will be so selected as to satisfy the boundary conditions:

$$\text{at } y = 0,\ u = 0;\ \frac{\partial u}{\partial y} \neq 0;\ \frac{\partial^2 u}{\partial y^2} = -\frac{UU'}{\nu};\ \frac{\partial^3 u}{\partial y^3} = 0;$$

$$\text{at } y = \delta,\ u = U;\ \frac{\partial u}{\partial y} = 0;\ \frac{\partial^2 u}{\partial y^2} = 0; \qquad (1)$$

the touch indicates the derivative with respect to x; $\nu = \frac{\mu}{\rho}$ is the cinematic coefficient of viscosity.

We choose the function $u(x, y)$, which in an approximate manner represents a family of velocity profiles in different sections of the boundary, in the form

$$\frac{u(x, y)}{U(x, y)} = \sin\frac{\pi y}{2\delta}\left[1 + \frac{2\lambda}{\pi^3}\left(1 - \sin\frac{\pi y}{2\delta}\right)\right]. \qquad (2)$$

Here $\lambda = \frac{U'\delta^2}{\nu}$ is Pohlhausen's parameter. It may easily be shown that the

function given by formula (2) satisfies the following boundary conditions:

$$\text{at } y=0,\ u=0;\quad \frac{\partial^2 u}{\partial y^2}=-\frac{\lambda}{\delta^2}U=-\frac{UU'}{\nu};\quad \frac{\partial^3 u}{\partial y^3}=-\frac{\pi^3}{8\delta^2}-\frac{\lambda\pi}{4\delta^2};$$

$$y=\delta,\quad u=U;\quad \frac{\partial u}{\partial y}=0;$$

$$\frac{\partial^2 u}{\partial y^2}=-\frac{\pi^2}{4\delta^2}-\frac{\lambda}{2\delta^2};\quad \frac{\partial^3 u}{\partial y^3}=0. \tag{3}$$

It follows from (3) that the function (2) satisfies all the boundary conditions of Pohlhausen with the exception of $\frac{\partial^3 u}{\partial y^3}=0$ for $y=0$ and $\frac{\partial^2 u}{\partial y^2}=0$ for $y=\delta$, which are not satisfied in the general form.

The expression (2) differs from Pohlhausen's expression for $u(x, y)$ [1] in enjoying the following essential property. The condition for the separation of the boundary layer is

$$\left(\frac{\partial u}{\partial y}\right)_{\substack{y=0\\ x=x_s}}=0. \tag{4}$$

In our case the condition (4) leads to the equality

$$\frac{\pi}{2\delta}+\frac{\lambda}{\pi\delta}=0,\quad \text{i. e. } \lambda=\lambda_s=-\frac{\pi^2}{2}\cong\geqslant 4.935. \tag{5}$$

This value of λ_s is fairly near to that obtained by experiment, $\lambda_s\simeq -5.37$. The probable reason for the satisfactory agreement with the experiment is that for $\lambda=\lambda_s$ the boundary condition $\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=0}=0$ is fulfilled, i. e. all of the inner boundary conditions are satisfied at the separation point. This brings out very clearly how important it is for defining the position of separation that the condition $\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=0}=0$ be fulfilled.

It is well to note that the family of velocity profiles proposed here can be used in the region of the boundary layer where the parameter λ varies within the limits

$$-\lambda_s>\lambda\geqslant\lambda_s. \tag{6}$$

It may be emphasized that for $\lambda=-\lambda_s$ the condition $\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=\delta}=0$ is fulfilled, which means that all of the outer boundary conditions are satisfied.

2. We shall use the integral relation of the boundary layer in the form

$$\frac{d\delta^{**}}{dx}=\frac{U'}{U}(2\delta^{**}+\delta^*)=\frac{\tau_0}{\rho U^2}, \tag{7}$$

where δ^* is thickness of displacement and δ^{**} is momentum thickness determined, respectively, by the relations

$$\delta^*=\int_0^\delta\left(1-\frac{u}{U}\right)dy;\quad \delta^{**}=\int_0^\delta\frac{u}{U}\left(1-\frac{u}{U}\right)dy. \tag{8}$$

τ_0 is friction stress on the boundary of the body.

In our case we have

$$\tau_0=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=\sqrt{\mu\rho U'U^2}\,\frac{b_1}{\sqrt{|\lambda|}}; \tag{9}$$

$$H=\frac{\delta^*}{\delta}=1-\frac{2}{\pi}+\lambda\frac{2}{\pi^2}\left(\frac{1}{2}-\frac{2}{\pi}\right)=0.3634+0.1366\frac{\lambda}{\lambda_s}; \tag{10}$$

$$H^{**}=\frac{\delta^{**}}{\delta}=\frac{2}{\pi}-\frac{1}{2}-\lambda\frac{2}{\pi^2}\left(\frac{14}{3\pi}-\frac{3}{2}\right)+\lambda^2\left(\frac{2}{\pi^2}\right)^2\left(\frac{8}{3\pi}-\frac{7}{8}\right)=$$

$$=-0.0262\left(\frac{\lambda}{\lambda_s}\right)^2+0.0146\left(\frac{\lambda}{\lambda_s}\right)+0.1366; \tag{11}$$

$$b_1=\frac{\pi}{2}+\frac{\lambda}{\pi}=\frac{\pi}{2}\left(1-\frac{\lambda}{\lambda_s}\right). \tag{12}$$

The values of H^*, H^{**}, b_1 for different values of λ/λ_s are tabulated below.

$\frac{\lambda}{\lambda_s}$	λ	H^*	H^{**}	b_1	f	F	τ_0^*
I	II	III	IV	V	VI	VII	VIII
1.0	—4.9348	0.5000	0.1250	0.000	—0.07713	0.9253	0.000
0.9	—4.4413	0.4863	0.1285	0.1571	—0.07333	0.8888	0.0745
0.8	—3.9478	0.4727	0.1315	0.3142	—0.06826	0.8465	0.1582
0.7	—3.4544	0.4590	0.1340	0.4712	—0.06203	0.7993	0.2537
0.6	—2.9609	0.4454	0.1359	0.6283	—0.05529	0.7480	0.3654
0.5	—2.4674	0.4317	0.1373	0.7854	—0.04651	0.6942	0.5000
0.4	—1.9739	0.4180	0.1383	0.9425	—0.03776	0.6399	0.6710
0.3	—1.4804	0.4044	0.1386	1.0996	—0.02844	0.5846	0.9040
0.2	—0.9870	0.3907	0.1385	1.2566	—0.01893	0.5306	1.2650
0.1	—0.4935	0.3771	0.1378	1.4137	—0.00937	0.4785	1.7895
0.0	0.0000	0.3634	0.1366	1.5708	0.00000	0.4290	∞
—0.1	0.4935	0.3497	0.1349	1.7279	0.00878	0.3838	2.4615
—0.2	0.9870	0.3361	0.1326	1.8850	0.01735	0.3399	1.8955
—0.3	1.4804	0.3224	0.1299	2.0420	0.02497	0.3105	1.6770
—0.4	1.9739	0.3088	0.1266	2.1991	0.03164	0.2760	1.5650
—0.5	2.4674	0.2951	0.1227	2.3563	0.03716	0.2511	1.5000
—0.7	3.4544	0.2678	0.1136	2.6704	0.04460	0.2182	1.4380
—0.8	3.9478	0.2541	0.1082	2.8275	0.04623	0.2099	1.4220
—0.9	4.4413	0.2405	0.1023	2.9846	0.04649	0.2052	1.4165
—1.0	4.9348	0.2268	0.0958	3.1416	0.04530	0.1954	1.4150

3. The integral relation of the boundary layer (7) can be transformed thus

$$\frac{df}{dx} = \frac{U''}{U'} f + \frac{U'}{U} F, \tag{13}$$

where

$$f(x) = \lambda H^{**2}, \quad F(x) = 2H^{**}[b_1 - \lambda(2H^{**} + H^*)]. \tag{14}$$

In practical calculations we shall use a simple method evolved by Loytzansky ([2], [3]) for the approximate integration of the equation (13).

The relation between F and f turns out to be nearly a linear one and can be represented as follows

$$F(f) = a - bf + \varepsilon(f). \tag{15}$$

Under condition (15), while $\varepsilon(f)$ is supposed to be known for the time being, one may integrate the equation (13) as linear equation to obtain

$$f = \frac{U'}{U^b} \int_0^x [a + \varepsilon(f)] U^{b-1} dx + c \frac{U'}{U^b}. \tag{16}$$

Here c is an arbitrary constant which may be determined from the condition for f at $x = 0$.

Equation (16) is integrated by the method of successive approximations. For first approximation we take

$$f_0(x) = a \frac{U'}{U^b} \int_0^x U^{b-1} dx + c \frac{U'}{U^b}. \tag{17}$$

The next approximation is

$$f_1(x) = \frac{U'}{U^b} \int_0^x [a + \varepsilon(f_0)] U^{b-1} dx + c \frac{U'}{U^b}. \tag{18}$$

In order for the first approximation to give results sufficient for practice in most cases, the value of $\varepsilon(f)$ should be small in comparison with a.

Put $a = 0.440$; $b = 5.85$. With these values of a and b the largest correction is $|\varepsilon(f)| \leqslant 0.04$.

When f is known, the value of λ is easily found from the table, and with it we find all the parameters characterizing the laminar boundary layer.

For determining the friction stress τ_0 the value of

$$\tau_0^* = \frac{\tau_0^\circ}{\sqrt{\mu\rho\,|\,U'\,|\,U^2}} = \frac{b_1}{\sqrt{|\,\lambda\,|}} \tag{19}$$

is also listed in the table.

It may here be remarked that with the adopted law of velocity distribution (2) a Pohlhausen's equation is obtained, and for its solution other available methods may be applied as well.

4. In order to evaluate the accuracy of the solution let us consider the case (3)

$$U = U_0(1-x) \tag{20}$$

under the condition that $f=0$ for $x=0$.

As the first approximation we get

$$f = -\frac{0.44}{(1-x)^{5.85}} \int_0^x (1-\xi)^{4.85}\,d\xi = -0.0735\,[(1-x)^{-5.85} - 1]. \tag{21}$$

Computations show that this approximate solution is in satisfactory agreement with the exact solution.

For the separation point ($\tau_0 = 0$, $f = -0.07713$) we get $x_s \simeq 0.114$, while the exact value is $x_s = 0.12$. For practical purposes this first approximation will suffice.

5. The function (2) which is suggested for the family of velocity profiles in different sections of the laminar boundary layer can serve to compute the laminar boundary layer of a body of revolution. In that case the integral relation of the boundary layer takes the form (4)

$$\frac{d\delta^{**}}{dx} + \left(2\frac{U'}{U} + \frac{r_0'}{r_0}\right)\delta^{**} + \frac{U'}{U}\,\delta^{**} = \frac{\tau_0}{\rho U^2}. \tag{22}$$

Here r_0 is cross-section radius. Otherwise the notation is the same.

Equation (22) is reduced to the form

$$\frac{df}{dx} = \left(\frac{U''}{U'} - 2\frac{r_0'}{r_0}\right) f + \frac{U'}{U}\,F. \tag{23}$$

This solution can also be solved by the method proposed by Loytzansky (1). Adopting a linear dependence between F and f and supposing f to be finite for $x=0$, we upon integration find

$$f = \frac{U'}{U^b r_0^2} \int_0^x [a + \varepsilon(f)]\,U^{b-1} r_0^2\,dx. \tag{24}$$

As first approximation one may be satisfied with the integral

$$f_0 = a\frac{U'}{U^b r_0^2}\,U^{b-1} r_0^2\,dx \tag{25}$$

where $a = 0.44$ and $b = 5.85$. For a body of revolution the characteristics of the laminar boundary layer are determined from the table. The separation point corresponds to $f = -0.07713$.

Central Research Institute
of the River Fleet.

Received
15. II. 1943

REFERENCES

[1] K. Pohlhausen, ZAMM, 1, 233 (1921). [2] L. G. Loytzansky, C. R. Acad. Sci. URSS, XXXV, 255 (1942). [3] N. E. Kochin and L. G. Loytzansky, ibid., XXXVI, 278 (1942). [4] L. G. Loytzansky, ibid., XXXVI, 183 (1942).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

PHYSICS

ON THE RELATIVISTICALLY INVARIANT «CUTTING-OF FACTOR» IN ELECTRODYNAMICS

By M. A. MARKOV

(Communicated by S. I. Wawilow, Member of the Academy, 19.V.1943)

Several different authors have suggested the use of relativistically invariant «cutting-off factors» ([1]) for resolving some well known difficult points in electrodynamics.

Their suggestions are briefly as follows: the vector potential describing the interaction between field and electron is written in the form of a Fourier series. A relativistically invariant function decreasing with frequency is introduced as a factor in every term of the expansion (the «cutting-off factor»).

If the problem is explored by means of the mathematical apparatus employed in electrodynamics of Dirac-Fock-Podolsky ([2]), one easily comes to conclude that the introduction of relativistically invariant cutting-off factors leads to an incompatible system of differential equations: the conditions for the integrability of a system of equations [Bloch's conditions ([3])] are disturbed.

The physical meaning of this circumstance is that in this case the Poisson brackets $[A(\bar{r}_s\, t_s),\ A(R,\ T)]$ have the shape

$$[A(\bar{r}_s\ \ t_s),\ A(\bar{R},\ T)] = \vec{\alpha} e_s \int \frac{1}{k} \sin\{\bar{k}(r_s - \bar{R}) - ck(t_s - T)\} D(k)\, dk_x\, dk_y\, dk_z, \quad (1)$$

if the cutting-off factor is taken in the form

$$D(k) \exp\left[-\sum_1^4 b_\mu k_\mu \right]. \quad (2)$$

In other words, the right-hand side of (1) is no longer a δ-function, and the values of the potential at points r_s and R are now not independent, even in case they refer to a single moment of time ($t_s = T$).

To put it differently, the relation (1) implies an infinite velocity of propagation of the signal.

The introduction of the cutting-off factor is equivalent to the consideration of the space charge.

If, in fact, the electrodynamics of D.-F.-P. is made use of, we can in any given system of coordinates obtain Dalambert's equation for the potentials. For the cutting-off factor (2), for instance, we have, provided the electron is at rest:

$$\nabla^2\varphi = -\frac{e}{2\pi^2} \int \cos\left\{ \bar{k}(r_e - R) \quad ck(t_e - T) \right\} \exp\left[-\frac{2mcbk}{\hbar} \right] dk_x\, dk_y\, dk_z =$$

$$= -4\pi \frac{e}{\pi^2} \cdot \frac{\gamma}{(\gamma^2 + r^2)^2} = -4\pi\rho \quad (3)$$

$$\text{for } t_e = T;\ \gamma = \frac{2bmc}{\hbar};\ r = |r_e - \bar{R}\ .$$

Here we are struck by the circumstance that the right-hand side of the equation, or the density of the charge, is always (i. e. in any system of coordinates) a function in space variables alone. The right-hand side of equation (3) has no sense in the D.-F.-P. electrodynamics unless in the case of transition to one common time ($t_e = T$) for the field (t_s) and for the particles (T), that is when the time simply drops out.

In other words, the frequently discussed «difficulty» which present the appearance of time in the expression for the charge density in Lorentz' transformations ([4]) is not actually a difficulty in itself. Nevertheless, the introduction of a relativistically invariant factor or, what is the same, of a space electron in the sense (3) is incompatible with relativism. It implies the existence of signals propagating at a higher speed that the critical velocity, and in this sense amounts to the consideration of a classically hard electron.

The result refers both to the quantum theory and the classical theory.

The classical electrodynamics in the D.-F.-P. form has been orderly treated by the author elsewhere ([5]).

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

Received
19. V. 1943.

REFERENCES

[1] G. Watagin, ZS. f. Phys., 88, 92 (1934). [2] Sow. Phys., 2, 468 (1932). [3] Sow. Phys., 5, 301 (1934). [4] W. Pauli, Hand d. Phys., 24, 271. (1933). [5] J. of Phys., VII, No. 1 (1943).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

CHEMISTRY

ON THE PROBLEM OF PRODUCTION OF ALITE CEMENT

By P. P. BUDNIKOV, Corresponding Member of the Academy, and
M. I. STRELKOV

Among the hydraulic binding materials two quickly setting and highly resistant types are known. One of them consists largely (up to 60—70%) of a low-basic calcareous aluminate $CaO \cdot Al_2O_3$ (1); this is the aluminium oxide cement. The other is essentially (up to 60%) composed by tricalcium silicate (alite), $3CaO \cdot SiO_2$; this is Portland cement.

The limestone or chalk, necessary for the production of these two types of cement, are everywhere available in sufficient quantities. Not so with the other ingredients of the raw material, namely, the one high in alumina, in the case of aluminium oxide cement, or in silica, in the case of alite cement. While there is a shortage of raw materials high in alumina, the reserves of raw materials high in silica (quartz sand, for instance) are practically inexhaustible.

It is known from the technology of the Portland cement that alite cement is very difficult to obtain by common methods, and the production of a cement with 95% of $3CaO \cdot SiO_2$ (alite) is even impossible. This is because a direct synthesis of tricalcium silicate from CaO and SiO_2 presents exceptional difficulties. Moreover, the alite thus formed is of inferior hydraulic qualities. The hydraulic qualities of a cement greatly depend upon the various anomalies in the structure of its crystalline lattice. In the presence of the liquid phase and by a long-lasting process a less active alite is obtained, than one formed in the solid state and during a shorter interval of time.

In synthesizing tricalcium silicate we employed the following method. First, $2CaO \cdot SiO_2$ was obtained, and then the requisite amount of lime or $CaCO_3$ was added. Afterwards the calcining was repeated at 1450—1500° C, during no more than 30—45 minutes. This method, involving reactions in a solid condition (2), is most likely to admit of the following schematic interpretation. When calcium oxide and silica enter into reaction in a mixture of any proportion, e. g. in the mixture presented by tricalcium silicate, the formation of $2CaO \cdot SiO_2$ proceeds at first rather energetically (3). Later on the process is complicated: on the one hand, $3CaO \cdot SiO_2$ is formed and, on the other, $3CaO \cdot 2SiO_2$. These compounds undergo further transformations: the ones higher in lime give it up to the less saturated compounds. The result is that at a certain moment all silicates of lime are present in the lime-silica mixture. On that account the compound $3CaO \cdot SiO_2$ is not obtained until the end of all possible re-groupings, and so its formation is delayed. If, however, we take a mixture containing no free silica and composed only by lime and bicalcium silicate, then none of the intermediate compounds is formed, and the ultimate product, $3CaO \cdot SiO_2$, is obtained at once. In this case the formation of tricalcium silicate proceeds at a strongly increased rate.

In our investigation we used the sand of Chasov-Yar (containing 99.89% of SiO_2) with chemically pure CaO and Al_2O_3. The materials were preliminarily reduced to a grain size that passed without remainder through a sieve of 4900 apertures per 1 cm^2. The raw material mixtures were made by the wet method in laboratory homogenizers. Calcining was carried out in molybdenum and cryptol furnaces, the curve of temperature rise corresponding to that usual for an industrial reverberating

furnace. The intermediate and the ultimate products were studied chemically and examined under the microscope. The free lime was estimated qualitatively by White's method and quantitatively by the phenol method (4). The clinker was examined in reflected light with the use of Insley's corrosive. The activity of the alite obtained by us was determined: a) from the rate at which lime passed from the powder into the water solution under perfectly comparable and reproducible conditions, b) from the resistance of the cement prisms (small specimens) to compression. In the latter case the indices obtained were compared with the corresponding indices for aluminium oxide cement, whose resistance to compression after 24 hours was found to be 430 kg/cm² (according to the standard method). In order to clear up the relation between the caking capacity and the activity of the alite clinker we determined the volume weight of the product of calcining.

The results of our studies can be reduced to the following main statements:

1) The intermediate product—bicalcium silicate—is easily obtainable by calcining a mixture of CaO or $CaCO_3$ and SiO_2. In that case the bicalcium silicate is obtained in the form of γ-modification and presents a fine powder gray in colour with a brown tint, which passes without remainder through a sieve of 4900 apertures per 1 cm². When the admixture of alumina and iron oxide is raised beyond 5—7%, and when the rate of cooling is increased, the amount of coarse fractions increases considerably.

2) An alite clinker of optimal qualities is obtained at 1500° C when the conditions of calcining are identical with those in a reverberating furnace. Such clinker is nearly one and a half times lighter than Portland clinker, and is very easily reduced to powder. The cement made of such a clinker hardened more rapidly during the first 24 hours in comparison with the alumina oxide cement, taken as a reference standard, which after 24 hours resisted a pressure of 430 kg/cm².

3) There exists a clearly expressed relationship between the caking capacity of the alite clinker (volume weight) and its activity: the higher the volume weight, the lower the activity. If the admixture of alumina (calculated in terms of tricalcium aluminate) is increased to 10% of tricalcium aluminate, the caking capacity of the clinker and its hardness increase continuously, while its activity shows a decrease. This phenomenon was traced by us up to a calcining temperature of 1700° C, and we found that, after 1550° C was reached, the decrease of activity proceeded at a lower rate. We expected $3CaO \cdot Al_2O_3$ to have a favourable influence on the caking capacity of an alite clinker, because it dissolves in $3CaO \cdot SiO_2$ in the solid state, but we failed to observe any such influence. In manufacturing alite clinker by the method of double calcining, our endeavour should be to reduce the amount of admixtures to a minimum.

4) When the calcining is carried on in a slightly reducing atmosphere, the alite clinker (cement) is green in colour. The stronger the caking, the more intense is this colour: the factors favourable for caking, favour also the intensity of the hue. The most strongly caked clinkers with an addition of alumina displayed the most intense green colour. In all cases the most intensely coloured side of the clinker was that in contact with the magnesite support in the furnace.

Our investigations have set up the main conditions for the production and the main properties of the alite cement. For a practical solution of the problem experiments on semi-industrial scale are needed, and the economic indices of the production of alite cement must be established.

Received
22. II. 1943.

REFERENCES

[1] G. Assarson, 'Reactions of aluminous cement with water. Symposium on the chemistry of elements, Stockholm (1938). [2] П. П. Будников и А. С. Бережной, Журн. прикл. химии, XIII, № 10, 1732 (1940). [3] П. П. Будников и Д. П. Бобровник, Труды 3-го Совещания по эксперимент. минералогии и петрографии при АН СССР, 223—229 (1940), Украинский химич. журнал, XII, кн. 4, 190 (1937). [4] Д. А. Ниренштейн, Зав. лабор., № 10—22, 1066 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

CHEMISTRY

SYNTHESIS OF SULPHANILAMIDE (STREPTOCID ALBUM)

By G. V. PIGULEVSKY

(Communicated by A. E. Arbusov, Member of the Academy, 11.II.1943)

The work of Gelmo([1]) on the synthesis of sulphanilamide has been taken as the basis for the technological scheme of the production of this compound, as soon as its medicinal importance was discovered.

In the USSR this process, worked out by Magidson and Rubzov ([2]), consists of the following operations; synthesis of N-acetylsulphanilyl chloride, production of the amide from the chloride, hydrolysis of N^4-acetylsulphanilamide with sulphuric acid, isolation of sulphanilamide by means of ammonia, and crystallization of the product from water.

On the basis of Gelmo's investigations the possibility of another technological approach can be foreseen, and such process has been worked out by the author.

The peculiarity of this process is in the stage of hydrolysis of N^4-acetylsulphanilamide, which is carried out by means of hydrochloric acid, and in the isolation of the crystalline sulphanilamide hydrochloride. Another characteristic feature of the process is the absence of crystallization of sulphanilamide either from water or from an organic solvent.

The details of the new technological process are as follows. N-acetylsulphanilyl chloride is synthesized, as described in the literature, from acetanilide and chlorosulphonic acid. The production of the amide from the chloride is carried out by means of an aqueous ammonium hydroxide solution. The N^4-sulphanilamide obtained is hydrolyzed with hydrochloric acid (1.08) taken in a threefold amount. Hydrolysis proceeds at the boiling point for 40 minutes.

When hydrolysis is complete, the mixture is cooled, sulphanilamide hydrochloride separates from the reaction mixture and is filtered off. Then it is dissolved in a small amount of water, and the solution is made colourless by treating it in the cold with activated charcoal. Neutralization of the colourless solution with an ammonium hydroxide leads to the separation of the crystalline sulphanilamide that is filtered off with suction, washed with water until a negative test for chloride ion and then dried. The yield of sulphanilamide (streptocid album) is 60% (calculated in terms of the acetanilide taken) *.

The sulphanilamide obtained, according to tests carried out by the Chief Drugstore Department of Leningrad, satisfies all the requirements of the pharmacopoeia.

Received
12. II. 1943.

REFERENCES

[1] Gelmo, Journ. f. pr. Ch, [2], 77, 372 (1908). [2] Магидсон и Рубцов, ЖОХ, 10, 756 (1940).

* From the technological process for the production of sulphanilamide, devised by the author, follows the possibility of purifying technical sulphanilamide, however contaminated. For one sample of the technical product A. A. Vansheidt and G. V. Pigulevsky have proposed a method of purification, based on the conversion of sulphanilamide into its hydrochloride, discoloration of the solution of the latter with activated charcoal and precipitation of sulphanilamide with ammonium hydroxide.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

CHEMISTRY

RECIPROCAL SYSTEM OF SODIUM AND BARIUM CHLORIDES AND NITRATES

By E. I. SPERANSKAYA

(Communicated by V. G. Chlopin, Member of the Academy, 9.II. 1943)

The ternary reciprocal system investigated, Na, Ba NO_3, Cl, is one of the simplest examples of a reciprocal system involving no formation of compounds, solid solutions or polymorphous transformations.

By its nature it is an intermediate type between reversible reciprocal and irreversible systems, reminding by its order of typical reversible-reciprocal systems:

1) Na, K $\| NO_3$, Cl [1]
2) K, Ca $\| NO_3$, Cl
3) K, Ba $\| NO_3$, Cl [2]

These systems are characterized by the formation of double compounds: $BaCl_2 \cdot 2KCl$; $KNO_3 \cdot KCl$; $CaCl_2 \cdot KCl$.

The fields of these compounds have the form of triangles and are in equilibrium with the fields of the components from which these compounds are formed at passage points (according to the terminology of A. G. Bergmann). Accordingly, these compounds are no components of eutectic mixtures in these reciprocal systems.

In reversible-reciprocal systems:

1) Tl, Pb $\| NO_3$, Cl [3]
2) Na, Pb $\| SO_4$, Cl [4]
3) K, Pb $\| SO_4$, Cl [5]
4) Na, K $\| NO_3$, J [6]

there are also long fields of compounds which, however, are wedge-shaped and join in two non-variant points, one or both of which are transition points. These double salts are participants of eutectic alloys of the corresponding phase triangles.

In the investigated system Na, Ba $\| NO_3$, Cl, as may be seen from Fig. 1 (a square), the surface of the liquidus consists of four fields of crystallization which join in two points.

Temperature	Solid phases	Liquid phases in mol. p. c.
A 280°	$NaNO_3 + NaCl + Ba(NO_3)_2$	7.5% Na_2Cl_2 + 11.5% $Ba(NO_3)_2$ + 81% $(NaNO_3)_2$
B 103°	$NaCl + Ba(NO_3)_2 + BaCl_2$	1% $(NaNO_3)_2$ + 38% Na_2Cl_2 + 61% $Ba(NO_3)_2$

Point *B* lies practically upon the diagonal $Ba(NO_3)_2$—Na_2Cl_2, possibly reaching sideways towards $(NaNO_3)_2$ by no more than 1—1.5%, thus occupying an intermediate position between irreversible and reversible-reciprocal systems.

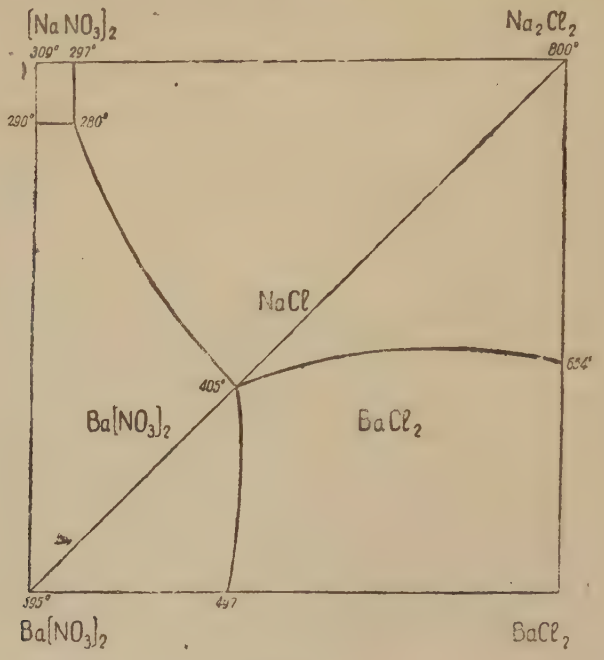


Fig. 1

If we turn to a series of other reciprocal systems of nitrates and chlorides, we shall find in the main sharply irreversible systems and singular systems with separation into layers:

1) Li, Na $\|$ NO_3, Cl [7]
2) Na, Tl $\|$ NO_3, Cl [8]
3) K, Tl $\|$ NO_3, Cl [9]
4) Ag, Tl $\|$ NO_3, Cl [10]
5) K, Ag $\|$ NO_3, Cl

The formation of compounds in the nitrate halide systems can be observed only on the stable diagonals in the following systems:

1) Hg, Tl $\|$ NO_3, Cl : $2TlNO_3 \cdot HgCl_2$; $Tl(NO_3) \cdot HgCl_2$ [11]
2) Hg, Tl $\|$ NO_3, Br : $TlNO_3 \cdot HgBr_2$ [11] (incongruent)
3) Ag, Hg $\|$ NO_3, I : $2AgNO_3 \cdot HgI_2$; $AgNO_3 \cdot HgI_2$ [12]

Thus, of the material on reciprocal systems of nitrates and halides discussed here, as well as of the data on other reciprocal systems investigated which number upwards of 100[13], the diagram Na, Ba $\|$ NO_3, Cl is an example of the simplest reciprocal system, being, besides, of intermediate nature between reversible and irreversible systems [14].

The reciprocal system Na, Ba $\|$ NO_3, Cl has been investigated also in aqueous solutions at 20° by Findlay and Cruickshank [5] and at 30° by Koppadora [6].

Between the diagram of melting of this system and the diagrams-isotherms at 20 and 30° there is considerable analogy in the character of the metathetic equilibrium.

At 20° the ternary monovariant point $Ba(NO_3)_2 + NaCl + BaCl_2 \cdot 2H_2O$ lies somewhat higher than the diagonal $Ba(NO_3)_2$—Na_2Cl_2, and at 30° somewhat lower. At 20° this point is a transition point, and at 30°—an eutonic point.

In other words, at 20° the system is reversible-reciprocal and at 30° irreversible-reciprocal. One should think that between these temperatures the system has an intermediate character.

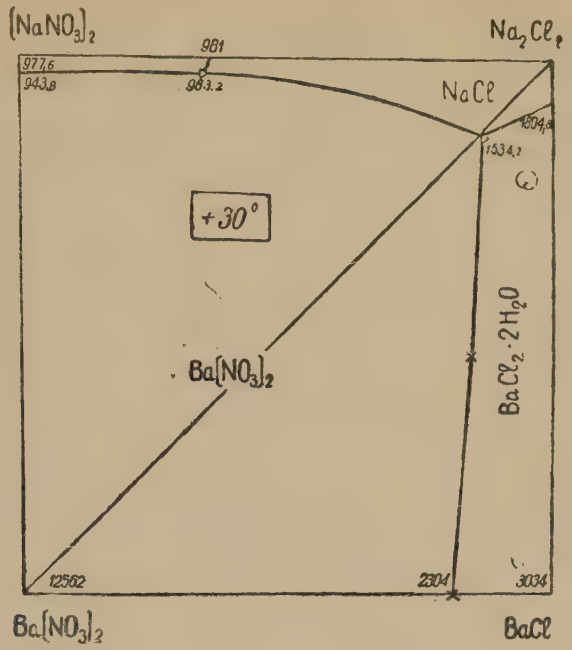


Fig. 2

An investigation of the solubility of this system at 0 and at 100° is under way in order to find out the influence of the temperature upon the shift of metathetic equilibrium in aqueous solutions.

Institute of General and Inorganic Chemistry.
Academy of Sciences of the USSR.

Received
9. II. 1943.

REFERENCES

[1] А. Г. Бергман, Справ. Техн. Энц., VII, 200 и 208 (1931). [2] А. П. Ростковский, ЖРХО, **61**, 89 (1929). [3] А. Г. Бергман и П. К. Леман, Труды Ростовск. гос. ун.-та, раздел химии (1941). [4] А. Г. Бергман, Справ. Техн. Энц., VII, 197 и 203. [5] Н. С. Домбровская, Известия сектора ФХА, XI, 13 (1936). [6] А. Г. Бергман и Ф. И. Васенин, там же, XI, 169 (1938). [7] В. П. Блидин, там же, XIII, 291 (1940). [8] Н. П. Лужная, Труды ГИПХ, вып. XXIII, 48 (1935). [9] А. Г. Бергман, Справ. Техн. Энц., VII, 197 и 204. [10] А. Г. Бергман и Т. А. Генке, там же, VII, 197 и 203. [11] А. Г. Бергман, Т. А. Генке и Ф. М. Исайкин, ЖРХО, **54**, 466 (1922). [12] А. Г. Бергман, ЖРХО, **53**, 181 (1921). [13] А. Г. Бергман, Известия Сектора ФХА, XIV, 105 (1941). [14] А. Г. Бергман и Н. С. Домбровская, ЖРХО, **61**, 1453 (1929). [15] A. Findlay and Cruickshank, Journ. Chem. Soc., 316 (1926). [16] Koppadora, Gaz. Chim. ital., **43**, 1, 138 (1913).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

GEOCHEMISTRY

PRESENCE OF MOLYBDENUM IN *LEGUMINOSAE*

By Kh. G. VINOGRADOVA

(Communicated by V. G. Chlopin, Member of the Academy, 13.II.1943)

In 1900 Demarçay ([1]) discovered molybdenum in the ash of trees qualitatively using the spectral method. He, however, inclined to explain away that molybdenum by contamination.

Later on molybdenum was reported in different, mostly vegetable organisms, by many investigators who also determined it qualitatively: by Cornec in 1919; by Forjas in 1929 and 1931; by Dongwald, Kibbin and Beans in 1934; by Konishi and Tsuge in 1936; by Sundara Rao in 1938.

Yet, it was not until 1931 that a series of systematic quantitative estimations of molybdenum was made by Ter-Meulen([2]) in different plants, soils, coals, etc. He was the first to call attention to the presence of comparatively high amounts of molybdenum in the seeds of plants, especially of *Leguminosae* and cereals.

The presence of molybdenum in vegetable organisms was also reported by Didier Bertrand in 1939 ([3]). His data furnished additional confirmation to the fact that the seeds of *Leguminosae* are relatively rich in molybdenum.

In a paper by Stanfield ([4]) that appeared in 1935 and discussed the content of molybdenum in soils, some data on the content of molybdenum in plants were also presented.

Isolated determinations of molybdenum in plants were referred to by Sundara Rao([5]) in his interesting work of 1940 on the occurrence of trace elements in plants. That author had made quantitative tests spectroscopically on several plants; on the dry matter basis he found the content of molybdenum in green peas to be 9.4×10^{-4} %.

One more indication to the high content of molybdenum in *Leguminosae*, especially in the green aerial parts of the clover grown on the pastures of Central Somerset, where the soil is exceptionally high in molybdenum, was made in connexion with the «teart disease» of cattle. There the cases of such affection among the cattle were ascribed to the unusually high quantities of molybdenum in the grass they had fed upon. On the other hand, plants are known sometimes to suffer from a deficiency of molybdenum in the soil ([6]).

In 1940 the present author started a study on the occurrence of molybdenum in soils, plants and animal organisms. The method I used is based on the colorimetry of the orange rhodanic complex. I have cleared up the possible errors involved in a quantitative estimation of molybdenum to which no due attention was paid before, for instance, the influence of salts of Fe, V, W, U, Pt and others on the colour of the rhodanic complex of molybdenum. I was able therefore to obviate the harmful effect of the above substances. My method will be described in detail in a special paper. By means of it as small a quantity of molybdenum as 5×10^{-7}g can be determined in a sample, and the latter therefore can be reduced in weight

down to several grams. As I found even traces of platinum to affect the colour of the rhodanic complex of molybdenum, I took care to eliminate any use of platinum vessels and to carry out the preparation of the material, including burning, solely in quartz dishes.

This article deals with a special problem, which is to evaluate the content of molybdenum in nitrogen fixing plants of the family *Leguminosae*.

Even from the brief, and probably incomplete, reviewal of literary data concerning the distribution of the molybdenum in plants, as made here, one comes to the conclusion that the content of molybdenum in *Leguminosae* is relatively high. This conclusion is also supported by our own data which are presented in Tables 1 and 2.

Table 1

Content of Molybdenum in Cereal Seeds from Different Localities of the USSR

Plants	Mo, wt % in dry matter	Origin of seeds
Wheat	9.62×10^{-5}	Transbaikalian
Wheat	2.71×10^{-5}	State Breeding Station of the TASSR
Oats	1.0×10^{-4}	Transbaikalian
Oats*	2.03×10^{-5}	State Breeding Station of the TASSR
Maize	4.38×10^{-5}	Kvareli
Maize	—	Latali
Barley	1.0×10^{-4}	Transbaikalian
Rye	7.3×10^{-5}	Transbaikalian

* Oats were gathered from the same plot with vetch and clover.

Table 2

Content of Molybdenum in Seeds of Different Leguminous Plants from Various Localities of USSR

Plants	Locality	Mo x 10^{-4}% by weight in dry matter
Phaseolus dark-brown	Kvareli, district Kvareli	14.0
» red-violet	» » »	12.0
» violet with mosaic	» » »	21.0
» mixture of varieties	» » »	6.1
» » » »	Eniseli » »	1.5
» dark-violet	Akhalsopeli » »	18.0
» light-brown	» » »	20.0
» brown with mosaic	» » »	17.0
» mixture of varieties	» » »	15.0
» dark-violet	Latali, Upper Svanetia	11.0
» light-brown	» » »	2.3
» brown with mosaic	Lakhamuli » »	0.9
» mixture of varieties	» » »	1.1
» » » »	» » »	1.8
» » » »	Bukistschihi, Chokhatauri district	2.6
Vetch	State Breeding Station of the TASSR, Kazan	1.3
» Lgov	Do	0.81
Peas Elite inexhaustible	Moscow	1.15
» Mandorf	State Breeding Station of the TASSR, Kazan	1.25
» Capital	Do	1.0
Lentil	Kazan, Market	0,9
» 4/105	State Breeding Station of the TASSR, Kazan	0.66
Blue lupen	Moscow	3.0

Another question which arises simultaneously with the study of distribution of molybdenum in organisms, soils and waters is that of the physiological rôle of this mineral. Of the numerous experiments planned to solve this problem, our attention was invited by those concerned with the influence of molybdenum upon nitrogen-fixing organisms. The first indications to the importance of molybdenum in fixing nitrogen were made in 1930 by H. Bortels. By now there is a considerable accumulation of papers devoted to these problems. The presence of molybdenum in the soil (medium) was reported to have a positive influence on the fixation of nitrogen by *Azotobacter*(7). Some new data point to the fact that molybdenum is a factor in the fixation of nitrogen by *Anabaena* (a symbiotic alga living upon the fern *Azolla*), in which, according to Ter-Meulen (8), the content of molybdenum is rather high.

So we may note three interesting facts: namely, the concentration of molybdenum in the seeds of *Leguminosae*, its concentration in *Azolla*, living in symbiosis with *Anabaena*, a nitrogen-fixer, and, finally, the promoting effect of molybdenum, when introduced in the medium, on the rate of fixation of nitrogen by plants, particularly by *Azotobacter*. In all of these three cases the rôle of molybdenum appears to be identical. In *Leguminosae* the fixation of nitrogen is known to take place through the agency of *Bact. radicicoli*, living in the nodules developing on the roots of plants. From the point of view just stated it was interesting therefore to estimate the content of molybdenum in the nodules of *Leguminosae*.

To make sure that the seeds of *Leguminosae* actually contain unusually high quantities of molybdenum, I undertook comparative tests for molybdenum in the seeds of different plants (Tables 1 and 3). In fact, the average content of molybdenum in the seeds of various plants is (from both literary data and ours) about $4.6 \times 10^{-5}\%$ (average of 25 analyses), whereas *L guminosae* contain about $5.54 \times 10^{-4}\%$ of molybdenum (average of 41 analyses).

To these data I may add those on the content of molybdenum in the cotyledons and axial parts of phaseolus (separated from Lakhamuli):

	Mo % in dry weight
Cotyledons	1.8×10^{-4}
Axial parts	4.1×10^{-4}

Yet I failed to obtain such a striking difference between the axial parts and cotyledons, as has been reported by Didier Bertrand.

However, as I have just mentioned, it was the content of molybdenum in the nodules of *Leguminosae* that I was mainly interested in, and in the literature I had found no data concerning this subject. For analysis I took nodules from three leguminous plants: vetch, lupen and clover, originating from different localities of this Union (entire plants were taken). In sampling a number of technical difficulties were encountered. In lupen the nodules had reached a rather considerable size, up to 10 mm in diameter, and could be comparatively easily separated from the roots, washed with distilled water and dried to constant temperature (+105°C). In vetch the nodules were very small, scarcely 1 mm across. The nodules of clover were somewhat larger, oval in form. Among the nodules of a single plant there was a wide variation of size. In order to obtain a sample of 1—2 g in dry weight, plants had to be collected and great care exercised in digging them up with the roots; the more so as it was essential in the case of vetch to preserve the young tiny rootlets where the nodules were mainly concentrated. It was hard to separate the nodules from the loamy soil and clean them. In the case of clover and vetch the nodules were not washed with water.

From the data of Table 3 it is quite clear that the nodules of *Leguminosae* are higher in molybdenum than is the soil, or the whole plant, or even the seeds.

In reality, the nodules of vetch and clover contain more molybdenum than is indicated in the table, because some contamination by the soil is not excluded, and in the case of vetch the admixture of minute rootlets, from which the nodules could not be absolutely separated, may also have had a certain influence.

Table 3

Comparative Content of Molybdenum in the Soil, in Nodules and in Plants

Plants	Mo, wt % in dry matter			
	Soil	Nodules	Whole plant	Seeds
Lupen (Moscow)	$< 10^{-4}$	1.7×10^{-3}	—	3×10^{-4}
Clover (Kazan)*	$< 10^{-4}$	1.1×10^{-3}	4.7×10^{-4}	—
Vetch (Kazan)*	$< 10^{-4}$	2.5×10^{-4}	—	1.3×10^{-4}

* Clover, vetch and oats were all collected from the same plot. The roots of vetch and oats were interlaced.

In this light it appears very interesting to explore the content of molybdenum in the nodules developing on the leaves of certain tropical plants, e. g. *Pavetta indica*, and also in those of *Cyanophyceae*, which are known to be nitrogen-fixers.

No less an interesting question is that of the rôle of molybdenum in plants—the nature of its participation in the chemical processes of nitrogen fixation. The more so as ammonia is considered by most of the workers as the first product of nitrogen fixation, and, on the other hand, oxidation processes are known to be accelerated catalytically by the presence of traces of molybdenum.

Experiments on molybdenum are under way.

W. I. Vernadsky Laboratory of Geochemistry Problems.
Academy of Sciences of the USSR.

Received
13. II. 1943.

REFERENCES

[1] Demarçay, C. R., **130**, 91 (1900). [2] H. Ter-Meulen, Recueil des Travaux chimiques des Pays-Bas, L. (4 série, T. XII), 491 (1931). [3] Didier Bertrand, C. R., **208**, 2024 (1939). [4] K. E. Stanfield, Ind. and Eng. Chem. Anal. Ed., **7**, No 4, 273 (1935). [5] Sundara Rao, J. Indian Chem. Soc., **17**, 351 (1940). [6] W. S. Ferguson, A. H. Lewis and S. J. Watson, Imp. Chem. Industr. Yealott's Hill Research St. Bull. No. I, 28 (1940). [7] H. Ter-Meulen, Nature, **130**, No. 329, 966 (1932).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

BIOCHEMISTRY

PROTEOLYSIS IN GRAIN AFFECTED WITH *EURYGASTER INTEGRICEPS*

By V. L. KRETOVICH, K. V. PSHENOVA and A. A. BUNDEL

(Communicated by A. N. Bach, Member of the Academy, 9.II.1943)

Kozmina and Markin (1), Blagoveshchensky and Sosedov (2), Kretovich and Tokareva (3) have established the main biochemical features of a grain affected with *Eurygaster integriceps*. The last of those authors were able to show by experiment the presence in an affected grain of powerful proteinases which are isolated from it in the form of dry enzymic preparations. The purpose of the present work was further to clear up the extent of proteolysis that occurs in a grain punctured by the insect, and to establish the nature of the proteinases contained in an affected grain.

It was proved by Tokareva (4), as well as by Goldstein, Gorbachevskaya and Sklovskaya (5) that the affected grains are distinguishable from normal grains by containing more free and peptid-bound tyrosine. In the light of Northrop's (6) work, who has demonstrated the capacity of the tyrosine to split off readily on the fermentative breakdown of proteins, this fact testifies to a profound proteolysis taking place in a grain affected with *Eurygaster integriceps*.

In order to characterize the affected grain in more detail, we made a quantitative estimation of tyrosine in a normal and in an affected grain by three different methods: according to Tsuverkalov, Thomas and Folin-Ciocalteau. The first two methods are not different in principle inasmuch as both of them are based on the response to Millon reagent. The difference was that in working according to Tsuverkalov, trichloroacetic acid was used for the precipitation of proteins, while in the tests carried on by the method of Thomas mercury acetate was used for that purpose. The method of Folin-Ciocalteau is essentialy different from the above two methods. As was found by us in special experiments, this method is by no means specific for the quantitative estimation of tyrosine, since the reagent of Folin-Ciocalteau gives a blue colour not only with tyrosine, but with triptofane, hystidine and cysteine as well, thus being a reagent for a whole group of aminoacids. It was therefore only natural to expect that the results of the estimations made according to the three methods would diverge greatly and they actually did so, as is obvious from Table 1.

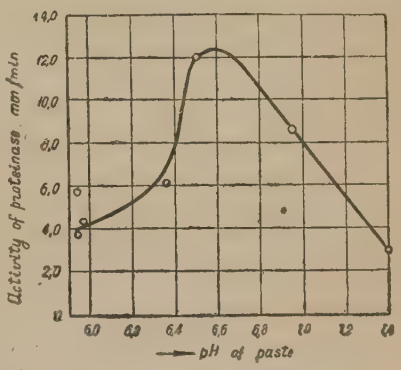
It may be seen from the tabulated figures that the difference in the content of tyrosine between normal and affected grains has been brought out with special clearness in the tests made according to Thomas. The methods of Tsuverkalov and Folin-Ciocalteau have yielded less representative data. This is because upon purifying the extracts of triacetic acid, a great deal of nitrogenous substances with a comparatively large molecule are left over in solution. In so far as these substances react with the reagents of Millon and Folin-Ciocalteau, considerable quantities of tyrosine are observed even in a normal grain. On the other hand, if the extracts

Table 1

Content of Tyrosine in Normal and Affected Grain, mg/g

Precipitation with CCl_3COOH				Precipitation with $Hg(NO_3)_2$		
Tsuverkalov		Folin-Ciocalteau		Thomas		
Normal grain	Affected grain	Normal grain	Affected grain	Normal grain	Affected grain	Affected portion of grain
0.344	0.512	—	—	0	0.038	—
0.500	1.128	1.00	1.56	0	0.043	—
0.604	1.800	1.11	1.59	0	0.056	0.087
0.580	1.728	1.01	1.62	0	0.058	0.078
—	—	1.04	1.50	0	0.062	0.071

are purified with mercury which gives a more complete precipitation of both proteins and the products of their disintegration, we find no longer any tyrosine in the extracts from a normal grain, and so the tyrosine, detected in the affected grain, does actually correspond to a free and peptid tyrosine, which is evidence of a deep fermentative breakdown of protein in the affected grain. Another evidence has been furnished by the analysis made by Kretovich and Tokareva who found the affected portion of the grain to contain non-protein nitrogen in especially large quantities.



So we have to give up the assumption that the disintegration of proteins in the affected grain is a result of the action of a particular decomposing enzyme. The data on tyrosine show plainly that the proteolysis is accompanied by energetic separation of amino-acids and peptids from the protein.

So profound a breakdown of the protein in the affected grain may easily be understood in the light of the data reported in our previous article ([7]). There we have shown that the proteolytic enzyme excreted by the salivary glands of *Eurygaster integriceps*, when it punctures the grain, belongs to the triptase group, since in the solution its action upon casein is a maximum at pH = 7.5—8.0.

At the same time it is known that the early stages of the action of proteinases (of triptase, in particular) are attended by disintegration of the protein which thereby changes its physical properties (e. g. its viscosity). A striking case in point is the alteration of physical properties in the gluten in response to the action of proteinase preparations or of extracts from a grain affected with *Eurygast r integriceps*. The baker is interested above all in these early stages of proteolysis, during which the decomposition of gluten takes place, and not in the final stages, which are determined by the accumulation of amine, carboxyl or phenol groups, and whose practical significance in the baking process is not so great.

The disintegration of gluten, indicated by the increase of its extensibility, has been placed at the basis of the new method for the determination of proteolytic activity, suggested by Kretovich and Tokareva ([8]).

In order to establish the most important characteristic of the proteinase contained in the affected grain, viz., its relation to the reaction of the medium, we had recourse to that method for the reason that it best meets the requirements of the baker, when the proteolytic activity of the grain or flour is to be estimated.

With this purpose in mind we investigated the action of a water extract (1 : 5) from the affected portions of grains upon gluten of normal flour at different pH of the dough. A paste, containing 10 g of flour (of a 70% yield), 3 cm^3 of

the extract and 3 cm^3 of a glycocoll buffer mixture with the respective pH was kneaded and left to stand at 20° C for 20 minutes. Afterwards gluten was washed off with water of 20° C in temperature and tested for extensibility.

Naturally enough, the actual reaction of the dough failed to correspond to the pH of the buffer mixture on account of the very high buffer capacity of the flour.

In order to determine the action of the dough, some portion of it was stirred in a half its weight of distilled water freed from CO_2, and in the solution thus obtained pH was determined electrometrically.

This method which is different in principle from the previously used ones, has furnished data that explain the favourable action of acidulation on a dough made of flour affected with *Eurygaster integriceps*. The acidulation of the dough not only inhibits that function of the proteinase which causes a profound splitting of the protein to aminoacids and peptids, but also its disaggregating function, which is the more important from the point of view of baking technology.

The results of these tests are presented in Table 2 and in the figure.

Table 2

Influence of pH of Dough upon Proteolysis of Gluten

pH of buffer mixture . .	6.10	7.80	9.70	1.07	12.67	13.07
pH of paste solution . .	5.94	5.96	6.36	6.50	6.95	7.45
Extensibility of gluten, cm/min . . .	37	44	61	120	86	31

It will be apparent that the optimum of the disaggregating action of the proteinase in an affected grain lies at pH = 6.6, nearly coinciding with the optimum of dissolving action of this enzyme upon gluten, as reported by Kretovich and Tokareva([8]).

To summarize the data at our disposal concerning the nature of the action of the proteinase contained in a grain affected with *Eurygaster integriceps*, we come to the conclusion that this proteinase belongs to the triptase group. So the function of this enzyme is to decompose protein, and deeply split the protein molecule, whereby an accumulation of triaminoacids and peptids is brought about.

Institute of Biochemistry.
Academy of Sciences of the USSR.

Received
19. II. 1943.

REFERENCES

[1] Н. Козмина и Г. Маркин, Сб. «Биохимия и микробиология пшеницы» (1934). [2] А. Благовещенский и Н. Соседов, там же, (1934). [3] В. Кретович и Р. Токарева, Биохимия, № 3, 387 (1938) № 4, 636 (1939). [4] Р. Токарева, Мукомолье и элеваторно-складское хозяйство, № 9 (1940). [5] Б. Гольдштейн, В. Горбачевская и О. Скловская, Біох мічный журнал, **16**, 193 (1940). [6] J. Northrop, Crystalline Ferments, 1940. [7] V. Kretovich, A. Bundel and K. Pshenova, C. R. Acad. Sci. URSS, XXXIX, No. 1 (1943). [8] V. Kretovich and R. Tokareva, ibid., XXVII, 571 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

HYDROBIOLOGY

THE MOLLUSC *CORBICULA FLUMINALIS* MÜLL. IN THE VALLEY OF THE MURGAB RIVER

By N. K. DECKSBACH

(Communicated by S. A. Zernov, Member of the Academy, 6.III.1943)

The distribution of *Corbicula fluminalis* Müll. in the USSR is no uniform one. The mollusc inhabits the lower course of the Kura river in Azerbaidjan, some regions of Kazakhstan and Uzbekistan; in the Far East it is found along the basin of the Amur and in the Maritime Province (1). L. S. Berg found *Corbicula* in the mouth of the Amu-Darya river.

The finding of *Corbicula* in Turkmenia, in the valley of the Murgab, serves as a link between all the places of Central Asia where the mollusc was encountered, pointing to a wider distribution of the species over Central Asia.

The presence of the mollusc in the Murgab river has until recently been refuted (2). In course of the work carried on in summer 1942 by a party of the Moscow University in the Murgab valley, I and A. L. Briukhatova ran across *Corbicula* at the following places of the lower reaches of this river: in the Yolotan water-reservoir, in the region of the Zonal cotton-lucerne station of the All-Union Cotton Institute, near the town Yolotan (within the system of the Sultan-Yab canal), in the river Murgab itself in the region of town Yolotan and in the 3rd Hindukush water-reservoir. Time of finding: May—August.

In the region of town Yolotan shells of *Corbicula* cover in huge numbers the alluvial sand, covering the bottom along the river banks; in the middle of the river the shells are met rarely if at all, the bottom being covered with lime. In the 3rd Hindukush water-reservoir the shells were found in great numbers along the river bank upon humid ground denudated after water drainage.

In the more elevated part of the Murgab valley, in the middle and upper courses (Tashkeprin and Sultan-Bent water-reservoirs, the course of Murgab river and the lakes of the inundation area between these two water-reservoirs, about 300 km long) *Corbicula* has not been encountered (data by F. F. Zentil vich).

The mollusc *Corbicula fluminalis* is of interest from the viewpoint of zoogeography. Along with the Indian species of *Limnaea luteola* (var. *oxiana*), *Melanopsis praerosa* and *Pseudamnicola* (species of the genus *Unio* being entirely absent); this species characterizes the whole Aralsk district (3).

Measurements of the length of the shells of the Murgab *Corbicula* gave the following figures: 1) 36 mm; 2) 30; 3) 28; 4) 28; 5) 30 (intact specimen, 2 valves); 6) 28; 7) 24; 8) 24; 9) 23; 10) 20 (intact specimen, 2 valves); 11) 22.5; 12) 23; 13) 20; 14) 20; 15) 20; 16) 18.5; 17) 17; 18) 17; 19) 17; 20) 12; 21) 36.5 (intact specimen, 2 valves); 22) 34 and 23) 34 mm.

The measurements of Murgab molluscs thus give data that entirely fit within those reported for the species in question, the size of the shell ranging from 20 to 37 mm.

The shells found were almost exclusively white, and it was no more than in 2—3 cases that we came across specimens showing violet tint inside of the shell. This certainly speaks of the fact that the organisms have died long ago. I have never succeeded in observing the molluscs alive; and only in a single case I found the body to stick to the inner wall of the shell, which was an evidence of the recent death of the mollusc.

The peculiarities of the hydrological regime of the river Murgab consist in a large quantity of alluvia and in a rather high degree of mineralization of the water which contains much Ca and SO_4.

The finding of *Corbicula fluminalis* in the Murgab river is not deprived of a certain zoogeographical interest. This mollusc has been found in fossil condition at the isle Chelepen under Old-Caspian deposits, containing *Cardium trigonoides*, and under Bakinskian strata. The age of the strata containing *C. fluminalis* should, evidently, be referred to the lowermost post-Pliocene. Since Chelepen lies within the area of the former discharge of Amu-Darya (Uzboy) into the Caspian Sea and taking into account the presence of *C. fluminalis* in the modern Amu delta, the place of finding is quite natural. The fact that *C. fluminalis* has now been discovered in the Murgab (Turkmenia) goes once more to confirm that this river belongs to the system of Amu-Darya-Uzboy, as is actually believed by geologists.

M. V. Lomonossov State University.
Moscow

Received
1. IV. 1943.

REFERENCES

[1] В. И. Жадин, Моллюски. Жизнь пресных вод СССР, 1 (1940). — [2] П. Викторов и С. Филиппов, Отчеты Всесоюзн. н.-и. ин-та озерного и речного рыбного хозяйства (1940). О. П. Антипова, там же (1939). [3] И. В. Старости, Определитель позвоночных животных Туркменской ССР, вып. 1, Рыбы (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

ECOLOGY

ON THE CONDITIONS AND PHASES OF THE SPRING TRANSITION OF *EURYGASTER INTEGRICEPS* TO ACTIVE LIFE AS OBSERVED IN SOUTH-WESTERN UZBEKISTAN

By K. V. ARNOLDI

(Communicated by A. A. Borissiak, Member of the Academy, 18.II.1943)

The literature dealing with *Eurygaster integriceps* contains nothing but rather general information concerning the terms of the spring awakening of the pest and the weather factors with which these terms could be brought into coincidence. Somewhat less scanty are the data for Europe, while those concerning Central Asia are extremely meagre. And yet the nature and terms of the spring awakening of *Eurygaster integriceps* do materially affect its subsequent activity, including the harm it causes; they should therefore be taken into account when looking for more rational procedures to be applied in order to control the pest.

The data here reported were obtained by the author while working in 1941—1942 with the expedition of the Academy of Sciences of the USSR in the Kashka-Darya department. In the basin of the Kashka-Darya the wintering of *E. integriceps* takes place over an immense area both in the valley and especially the mountains, at an altitude of 500 to 3000 m over the sea level. It is therefore only natural that the hibernation of the insect proceeds under varied conditions, and that the bugs become active at different periods.

A relatively small number of bugs (as compared to that hibernating at mountainous localities), though yet a rather considerable one, pass the winter at a height of about 600—700 m in the Kashka-Darya valley. The bugs hibernate there in oases, on the carpet of gardens and vineyards, in the sod of cereals, along irrigation systems. In a state of torpidity the bugs lie in a medium, saturated with moisture. According to the data of the local meteorological station Kitab; 80% of the precipitations fall out during winter and two spring months (424 mm). Within this period the relative humidity fluctuates from 50 to 65%, the annual medium being 41%. It is only during January that negative temperatures are recorded (—2.2°); the snow covering is formed occasionally and is preserved during no more than 2—3 days. On the other hand, winter and spring are characterized by sharp temperature fluctuations which sometimes drops as low as—10°, this being usually accompanied with snowfalls. From measurements of temperature taken for air and soil, as well as from measurements of the moisture of the air layer adjacent to the soil at the places of hibernation of *Eurygaster integriceps*, the two following main observations can be made.

1) Under the shelter of the snow covering, which is usually formed at the periods of sharp temperature minima, the temperature but rarely falls below zero; this is the more true for the dead leaf and grass carpets of gardens. 2) The water-saturated medium around the insect shows a great heat capacity. Owing to this, the heat absorbed by the surface of the carpet (up to 20° C) during day is not appreciably transmitted into its thickness and fails therefore to affect the

bugs. During the early period of spring (second half of February) the 24-hour-day-means are 2—4° C, and at midday the temperature of the air does not exceed 10—14°; this period is characterized by the vegetation of some winter plants, and by the flowering of *Sternbergia Fischeriana* and *Gagea Capusii* and *G. graminifolia*; this period has no appreciable influence upon the bug either.

In 1942 the temperature sharply rose towards the end of February, having reached 9.2° C during the last three days. Though most of the bugs still remained in a state of torpidness in their wet layer of the carpet which never got warmer than 6° C, separate individuals already reached the very top layer of dry leaves whose external surface was heated up to 26° C, where they enjoyed a temperature of 17°C. When disturbed they at once passed to an active state. It is evident that these individuals awoke from their torpidity and migrated to the warmest layers. It is therefore the 27th of February that should be considered as the beginning of the spring activity of *Eurygaster integriceps* in 1942.

The moment when the bugs awake from their state of torpidity coincides with the period when the mean 24-day temperature is established at about 5.5—6° C. At first those of the insects become active that lay in a more superficial layer of the moist carpet where the temperature attains 7—7.5°. The microclimate of the wintering place, as connected with the microrelief, plays here the principal part. At this first phase of their activity the bugs do not leave their winter abodes; they only remove from one place to another and start sucking the etiolated sprouts and twigs of various plants. According to D. M. Fedotov's observations, this is seen from the fact that by this time their medium intestine is found to contain particles of fresh food. The posterior intestine is still distended by liquified fecalia, which is typical of the winter period.

The second phase is marked by the appearance of the bugs at the day surface. Sharp temperature fluctuations and precipitations that took place within the first 5 days of March have delayed the awakening of the bugs. The next 5 days proved warmer: on the 5th of March the mean temperature attained 8° C, on the 6th—9.5°, while the maximum was 16° at a 70% air humidity. This was the day when the bugs were first observed at the surface of dry leaves, though the great majority of them still remained in their abodes. And it was not until the second period of colds, followed by a more stable mean temperature of about 9.5—10°, that one could observe them to leave their shelters in huge numbers (towards the end of the first fortnight of March). Thus, the first phase lasted 11—12 days, after which the bugs passed to the 2nd phase, that of active migrations, short flights and feeding on green vegetation. It is this period that is usually pointed out in literature as the «awakening» of *Eurygaster*. As may be seen from what has been said above, this is not true.

During the 2nd phase *Eurygaster integriceps* undergoes a number of internal changes that have been recorded by D. M. Fedotov. Their posterior intestine becomes evacuated and flattens out (which is a condition indispensable for flying), green food appears in the medium intestine, the ovaries begin to develop, etc.

The first flights of the bugs were observed on the 14th and 16th of March, at a temperature of 20.2° C. They were creeping on the ground; separate individuals could be seen on young rosettes of *Carduus* and on the stalks of *Poa bulbosa*; it is on this plant that the first cases of sucking have been observed. However, variability of temperature and general heat deficiency, prevailing within this period, prevent the bugs from passing in large numbers to green food. The 2nd phase is thus extended to 17 days.

The end of the 2nd phase coincides with the setting in of temperature means, ranging from 14.5 to 15°C, which takes place towards the end of March. The bugs start feeding on green vegetation, and somewhat later, to multiply. The beginning of the 3rd phase coincides with the most pernicious period of the injurious activity of the insects. The spring phenology of the transition of *Eurygaster integriceps* to active life is well illustrated by the table below.

In the belt of foothills the dates of the activization of *Eurygaster* do but little differ from those observable in the valley. In the belt of low mountains, however,

Phases	Mean temperature	Date	Number of days	Biological indices
I	6.5—9.0°	27.II—11.III	12	Blossoming of *Sternbergia*, mass flowering of dandelions
II	9.5—14.5°	12.III—28.III	16	Profuse tillering of winter crops; flowering of plums and *Muscari*
III	above 14.5°	from 28.III	—	Flowering of pear, greening of *Crategus* and rose; end of vegetation of *Sternbergia* and early *Gageae*

the peculiarities of heat distribution; due to the exposure of the slopes; are of great importance, and the general delay in the transition of the bugs to active life amounts to 10—12 days. At a medium altitude, usually chosen by *Eurygaster* for its wintering places, the peculiarities of mountainous climate and microclimate play a still more important rôle, and the whole process is still more extended. In the middle of April, when the temperature was up to 10° C at day time, and the soil was heated to above 20° (while the minima were as low as 0), the bugs were found at those altitudes to be in the 2nd phase, while near the remainders of consolidated snow and below them they remained in a state of total torpidity at temperatures ranging from 0.7 to 3° C. The shrubs around were entirely devoid of leaves on April 21; the buds of *Acantholimon* scarcely began to disclose. At drier places the cereals started germinating, *Tulipa turkestanica* and *Corydalis* were in buds, *Colchicum luteum* was flowering near the snow accumulations, among the *Juniperus* shrubs *Gagea vegeta*, *G. emarginata*. *G. Capusii* were also flowering. Most insects were in full activity. Until the 9th—11th of May at altitudes of 2200—2500 m part of the *Eurygaster* bugs are still lying under the snow in hibernal state. Some of the insects quite nearby may be seen to start feeding and copulating. Heat deficiency of the air prevents the downward migration of these insects. This would, besides, be of no use, since such a belated flight would not give them the possibility to achieve propagation before the harvesting of the fields and the withering of the vegetation down in the valley.

The whole period of the transition of *Eurygaster integriceps* to active life in the Kashka-Darya valley is thus extended over no less than 70 days. This fact is of paramount importance, for it enables the bugs to descend into the valley at various periods during a long time. According to D. M. Fedotov, the new lots of bugs, descending from the mountains, may easily be distinguished by a poor development of their egg-tubes, their under-fed condition, etc.

The grave damage, caused by the *Eurygaster* bugs that have passed winter in the valley, makes us focus upon them our most serious attention. The knowledge of the conditions and stages of the spring awakening of the bugs, as well as phenological observations; carried out according to the scheme suggested above, will make it possible to improve the existing methods of advertisement. In those cases when the wintering places situated in the valley failed to be duly treated during winter, particular attention ought to be paid to the 1st phase of activity; at this period it is the mechanical methods of control that are the most expedient. To these belong the scraping and loosening of the upper ground layer, made with the purpose of exposing the bugs to the effect of critical temperatures; as well as local irrigation and the feeding of poultry upon the respective plots. Contrary to this, irrigation practised during winter; when the bugs are in a state of torpidness, is less promising.

Lastly, a considerable delay in the transition of the bugs to an active state at important altitudes may prevent their downward migration, and quite unexpectedly put under menace crops sown high in the mountains. In this case, advertisement made at due time is necessary.

A. N. Sewertzoff Institute for Evolutionary Morphology.
Academy of Sciences of the USSR.

Received
5. III. 1943.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 1

ECOLOGY

ECOLOGY AND DISTRIBUTION AREAS OF SOME *SCARABAEIDAE* SPECIES

By A. S. KOSMACHEVSKY

(Communicated by A. A. Borissiak, Member of the Academy, 18.II.1943)

In the case of *Scarabaeidae* the study of the behaviour of the organism in relation to its surroundings is greatly furthered by gaining an adequate knowledge about the length of a generation (3 to 5 years), the vertical migration of the larvae in the soil, and about the flight of the beetle as dependent on its body temperature and crepuscular illumination. Many of the representatives of *Scarabaeidae* inflict great harm upon agriculture and forestry, and the solution of the problems referred to above may be turned to profit, when working out measures to be taken against them.

It has been reported in literature that the development of *Melolontha hippocastani* F. varies in length from 4 to 5 years ([1]) in different regions of this country. But the effect of temperature on the rate of development of this beetle has not yet been studied experimentally.

The author's investigations of the May-beetles were carried on in the course of several years, both under field and laboratory conditions, at various regions of the USSR (Leningrad, Voronezh, Dnepropetrovsk and Kiev provinces, Krasnodar department and in the Kirghiz SSR). The following *Scarabaeidae* species were subjected to investigation: *Polyphylla irrorata* Gebl., *Polyphylla adspersa* Mots., *Melolontha hippocastani* F. (the eastern May-beetle), *Melolontha melolontha* L. (the western May-beetle), *Amphimallon solstitiale* L. (the June-bug). Upon hatching the larvae were placed one by one into tea glasses filled with humid soil (20—35% total moisture capacity). Every 4th and 7th day both soil and food were renewed, and every fortnight or month the larvae were weighed. Temperature measurements were taken daily.

The whole developmental cycle of *Melolontha hippocastani* F. from the egg to sex maturity was found to be achieved within 310 days, the total of effective temperatures (above 9—11° for different stages) made up 3740°.

The developmental cycle of *Polyphylla irrorata* Gebl. requires a total of effective temperatures of 6000°, that of *Polyphylla adspersa* Mots.—6200°, while the June-beetle (*Amphimallon solstitiale* L.) needs no more than 2788°.

The eastern May-beetle is distributed in Northern and Middle Europe, in the northern, central and partly in the southern part of the USSR, in Siberia (up to the Transbaikalia), and in Asia Minor. According to data by A. P. Semenov, in Transbaikalia this species forms a local variety, *Melolontha hippocastani* var. *baicalica* Reitt.

In the North (Leningrad, Kalinin, Ivanov provinces) the generation of the May-beetle is a five-year-long one, while in the South (southern part of the Voronezh and Kuibyshev provinces) the generation lasts no more than four years.

In the South-West, beginning from the line (more strictly speaking band) Pskov—Vitebsk—Smolensk—Kharkov, the eastern May-beetle is joined by a related species, the western May-beetle. The area of distribution of the latter extends over Middle and Southern Europe, West and South of the European part of the USSR, northern part of the North Caucasus (2).

Experiments carried on under laboratory conditions have shown the developmental rate of May-beetles to be dependent on temperature. From a comparison of the data obtained and the sum of the effective temperatures of the soil; as taken during the period of growth at various regions, it is possible to explain the length of generation. In the Voronezh province (Khrenovoe) the total of the effective temperatures of the soil within a summer season taken at a depth of 10—20 cm make up about 1000°; in the neighbourhood of Leningrad they make up 750-800°. Consequently, at the Voronezh province the generation of May-beetles will last 4 years, while in the Leningrad province it will be completed within 5 years.

One of the most important questions to be solved when studying the distribution of the western May-beetle is, Why this species does not spread eastward beyond the line Pskov — Vitebsk — Smolensk — Kharkov?

Let us analyse some comparative ecological data concerning the western and eastern May-beetle, as given in the table.

Ecological data	*Melolontha melolontha* L. (western May-beetle)	*Melolontha hippocastani* F. (eastern May-beetle)
Crepuscular flight begins at an illumination (at sunset 100%)	2—3%	60—95%
Body temperature at the start of flight .	31°	25°
Air temperature at the start of flight . .	8—9°	7.5—8.5°
Habitats of larvae	Fields and outskirts of forests	Mainly forests
Length of generation	3—4-year-long	4—5-year-long
Moult of larvae to second age takes place	At the year of egg-laying	At the year following oviposition
Larvae of 1st—2nd age hibernate at a depth of	20—50 cm	40—80 cm
Temperature limit of egg development . .	11—30°	11—24°
Number of days with a mean 24-hour $t°$ below 10°, recorded at the areas of occupation	No more than 30	60—80
Soil $t°$ at a depth of 40 cm drops to . .	Not below —2°	—5° and below

From a comparison of these data it may be inferred that *Melolontha melolontha* is a more thermophilic species: the upper threshold of egg development is by 6° higher than that of *M. hippocastani*, and soil temperatures rising up to 24—27°C do not make the larvae migrate downward. They continue feeding and developing in the upper layer till the autumnal decrease of temperature, when they conceal themselves deep in the soil, and in spring are the first to attain the upper layer. This species inhabits unsheltered places which are more exposed to the sun. All these factors favourably affect the rate of its development: the generation of *M. melolontha* lasts 3—4 years, which points to a better adaptation to using up heat.

M. hippocastani occupies more sheltered habitats, its larvae hibernate at a greater depth, they keep away from temperatures above 19—24°, showing a certain periodicity in their development. As a result, the development is somewhat delayed in this species, the generation is a 4—5-year-long one. But they are better adapted to the severe conditions of wintering: they can withstand lower soil temperatures, which allows them to spread eastward far beyond the limit of distribution of *Melolontha melolontha*, the western May-beetle.

Hibernating at a small depth (20—50 cm) the larvae of the latter, after having freshly moulted into the second age, perish at those places where the winter

temperature of the soil drops at this depth down to — 5° and below. Consequently, the spreading of *M. melolontha* to the East is mainly prevented by the low winter temperature of the soil, which results in the freezing out of the larvae of the second age, which do not descend deep for hibernation.

The *M. melolontha* beetles that start flying some 30—40 minutes later than those of *M. hippocastani*, are not so liable to be destroyed by predatory animals, active at early twilight (frogs, birds, hedgehogs etc.), but they cannot spread far to the East, where temperature drops rapidly after sunset (down to 9° and below), which makes the conditions unfavourable for the flight of the beetles ([3]).

It is interesting to consider the possible ways of evolution of the two species of May-beetle. A study of the ecological peculiarities of these two closely related species brings us to understand the mode of formation of their adaptive characters. We deal here with two species which are very close as to their morphological characters; at the larval stage they cannot by differentiated, though they vary greatly as to their physiological characters.

The two species are undoubtedly closely related and have arisen divergingly in the process of historical development. We believe the western May-beetle, *Melolontha melolontha* L., to be the older of the two species, and the eastern May-beetle, *M. hippocastani* F., to have diverged from it. The arguments in favour of this assumption are the following: *M. melolontha* is more thermophilic, it is distributed mainly over Western Europe and does not spread to the East beyond the line Pskov — Vitebsk—Smolensk—Kharkov; it is less adapted to seasonal changes, its generation is shorter (3—4 years). It tends to a continuous development, and, consequently, sharp seasonal changes lead to a mass destruction of its larvae. The area of distribution of this species could have been a wider one at the time when the climate was milder and showed no abrupt changes.

Unlike this, the eastern May-beetle, *M. hippocastani* F., shows a certain adaptation to the seasonal mode of life: larvae of the 1st—2nd age, after having reached in summer a definite stage of development, do not remain in the upper layer where they would undergo meult, but dig themselves deeper into the ground and do not moult until next year. This is why they can descend to deeper soil layers unaccessible to frosts, and are more cold-resistant as compared to the freshly moulted larvae of *M. melolontha*. Owing to its better adaptation to winter colds, *M. hippocastani* is able to spread far to the East, reaching Irkutsk and even farther. At the same time this species inhabits also Western Europe. It stands to reason that *M. hippocastani*, which has a wider area of distribution and shows a higher adaptability to unfavourable conditions, is the youngest of the two species.

The study of the two closely related species makes it possible to trace the differences between their physiological characters, and to bring out the significance of the adaptive peculiarities for the existence of the species.

Department of Zoology and Entomology.
Kirghiz Agricultural Institute.

Received
5. IV. 1943.

REFERENCES

[1] В. М. Березкина и В. И. Старк, Итоги н.-и. работ Всес. ин-та защиты растений за 1935 г., (1936). [2] Список вредных насекомых СССР и сопредельных стран, Труды по защите растений, 1 серия, вып. 5 (1932). [3] А. С. Космачевский, Зоолог. журнал, XX, вып. 2 (1941).